Stabilization of Load Voltages in Power Supply Systems with Limited Capacity Voltage Sources

Penin A., Sidorenko A.
"D. Ghitu" Institute of Electronic Engineering and Nanotechnologies of the Academy of sciences of Moldova Republic
aapenin@mail.ru, anatoli.sidorenko@kit.edu

***Abstract* — Restricted load powers, two–valued regulation characteristics, and interference of several loads is observed in power supply systems with limited power of voltage source. In this paper a geometrical approach is presented for interpretation of changes or "kinematics" of load regimes; the definition of load and regime parameters is grounded. Non-Euclidean geometry is a new mathematical apparatus in the electric circuit theory, adequately interprets "kinematics" of circuit, and proves the introduction and definition of the proposed concepts.**

***Key Words* — Load influence, projective transformation, regulation characteristic, voltage source.**

I. INTRODUCTION

The limitation of load powers, two-valued regulation and load characteristics are appeared in power supply systems with limited capacity voltage sources. If such a power supply contains some quantity of loads with individual voltage regulators, the interference of loads takes place [1]. Distributed, autonomous or hybrid power supply systems with solar cells, fuel elements, and storage energy modules (battery, ultra capacitor) can be examples of such systems [2].

Therefore, it is necessary to obtain relationships for the definition of the regime and regulation parameters, for example, at a possible coordinated predictive control for preset load regimes [3].

In the present paper, the main results of interpretation of changes or "kinematics" of load regimes [4] are discussed.

II. ANALYSIS OF VOLTAGE STABILIZATION REGIMES OF LOADS

Let us present the necessary results [4]. To do this, we consider a power supply system in Fig.1. The power supply system includes two idealized voltage regulators VR_1, VR_2, and load resistances R_1, R_2. The regulators define the transformation ratios n_1, n_2. At the same time, the internal resistance R_i determines the interference of the regulators on load regimes.

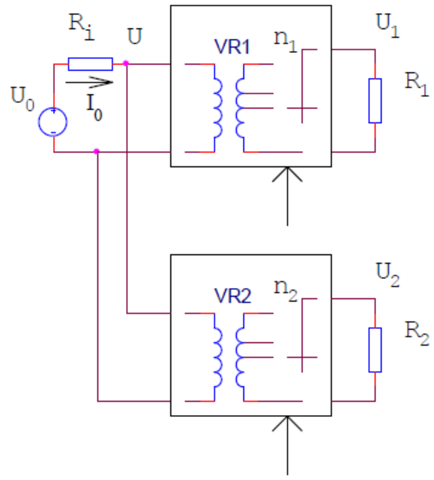

Fig.1 Power supply system with two voltage regulators VR_1, VR_2 and loads R_1, R_2.

A. *Case of one load*

In this case, the transformation ratio $n_2 = 0$. Let us obtain an equation describing the behavior or "kinematics" of this circuit at change of the parameter n_1. By definition,

$$n_1 = U_1 / U . \quad (1)$$

Load power

$$P_1 = \frac{U_1^2}{R_1}, P_1 = UI_0 = U\frac{U_0 - U}{R_i}.$$

Then, $\frac{R_i}{R_1}U_1^2 + U(U_0 - U) = 0$.

It now follows that

$$\frac{R_i}{R_1}U_1^{\ 2}+\left(U-\frac{U_0}{2}\right)^2=\frac{U_0^{\ 2}}{4}. \qquad (2)$$

For different values R_1 (i.e. R_1^1, R_1^2 and so on), this expression represents a bunch of circles (ellipses) in Fig.2.

Let a stabilized load voltage $U_{1=}$ be given. Then, the vertical line with coordinate $U_{1=}$ intersects the bunch of circles in two points generally. At the same time, the transformation ratio or variable n_1 is resulted by a stereographic projection of circle's points from the bottom pole $0,0$ on the tangent line at the upper pole [5], [6]. For example, the load R_1^1 corresponds to the variable n_1^1. For minimum value of load resistance $R_{1\min}$, the circle is tangent to the vertical line and voltage $U=0.5U_0$.
Using (2), we get the minimum value of the load resistance

$$R_{1\min}=4R_i(U_{1=})^2/U_0^{\ 2}. \qquad (3)$$

Also, the respective maximum allowable transformation ratio

$$n_{1\max}=2U_{1=}/U_0. \qquad (4)$$

The operating area of all the circles must be above the diameters of these circles; that is, voltage $U \ge U_0/2$. But, on some step of switching period at increase of the parameter n_1, a running point can pass over the diameter that is inadmissible. So, we must decrease the next values n_1 by some rule. In this sense, we come to hyperbolic geometry.

Let us consider the functional dependence $R_1(n_1, U_{1=})$, where the voltage $U_{1=}$ is a parameter. Similarly to [4], we must validate the definition of regime and its changes; find the invariants of regime parameters.

To do this, we consider such a characteristic regime as $R_1=\infty$. In this case, the ellipse degenerates into the two straight lines, $U=0$, $U=U_0$.
Then, for the voltage $U=U_0$, the transformation ratio

$$n_{1\infty}=U_{1=}/U_0. \qquad (5)$$

However, the question arises about the range of transformation ratio as $0<n_1<n_{1\infty}$. According to Fig.2, this range corresponds to the negative load value $R_1<0$ and expression (2) determines a hyperbola. In this case, the load gives back energy and the voltage source U_0 consumes this energy.

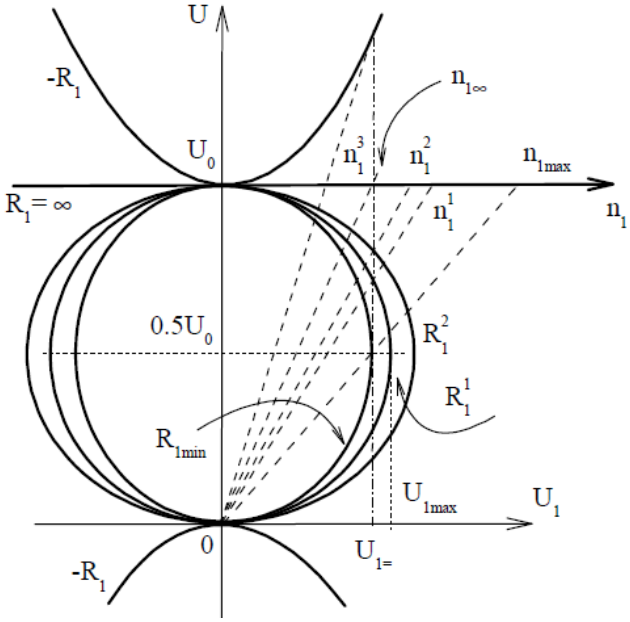

Fig.2 Stereographic projection of bunch of the ellipses $U(U_1, R_1)$ on the line n_1.

Then, it is possible to connect up the voltage source U_1 instead of the resistance $-R_1$. At the same time, the input resistance of this circuit must be equal to the constant value R_1 at change of the voltage value U_1. Such a condition is satisfied due to the variable value n_1; there is a loss-free resistance [7], a high-power-factor boost rectifier [8].
Taking into account (1), (2), we obtain

$$U_1=\frac{n_1U_0}{1+\frac{R_i}{R_1}n_1^{\ 2}}. \qquad (6)$$

Thus, we get the required relationships $n_1(R_1)$, $R_1(n_1)$

$$n_1^{\ 2}-n_1\frac{U_0}{U_{1=}}\frac{R_1}{R_i}+\frac{R_1}{R_i}=0,\quad \frac{R_1}{R_i}=\frac{n_1^{\ 2}}{n_1\frac{U_0}{U_{1=}}-1}, \qquad (7)$$

The dependence $R_1(n_1)$ determines a hyperbola in Fig.3. We have a single-valued mapping of hyperbola points on the axis n_1. This projective transformation preserves a cross ratio of four points [9]. Similarly to [4], [5], let us constitute the cross ratio m_n^1 for the points $0,\ n_1^1, n_{1\infty}, n_{1\max}$

$$m_n^1 = (\,0\ \ n_1^1\ n_{1\infty}\ \ n_{1\max}\ ) = \\ = \frac{n_1^1 - 0}{n_1^1 - n_{1\max}} \div \frac{n_{1\infty} - 0}{n_{1\infty} - n_{1\max}}\,. \tag{8}$$

The points $0, n_{1\max}$ are base ones and point $n_{1\infty}$ is a unit one. The point n_1^1 is a point of an initial or running regime.
Using (4), (5), we get

$$m_n^1 = \left(0\ \ n_1^1\ \ \frac{U_{1=}}{U_0}\ \ 2\frac{U_{1=}}{U_0}\right) = \frac{n_1^1}{n_{1\max} - n_1^1}\,. \tag{9}$$

In this case, the value m_n^1 is a non-uniform coordinate of the value n_1^1.

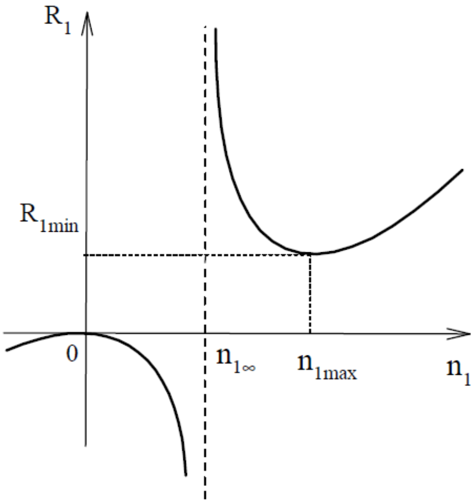

Fig.3 Hyperbola $R_1\,(n_1)$.

Further, the cross ratio m_n^{21}, which corresponds to a regime change $n_1^1 \to n_1^2$, has the form

$$m_n^{21} = (\,0\ \ n_1^2\ n_1^1\ n_{1\max}\ ) = \frac{n_1^2(n_1^1 - n_{1\max})}{n_1^1(n_1^2 - n_{1\max})}\,. \tag{10}$$

Using normalized values $\overline{n}_1^2 = n_1^2 / n_{1\max}$, $\overline{n}_1^1 = n_1^1 / n_{1\max}$, we get regime change (10) in the view

$$m_n^{21} = (\,0\ \ \overline{n}_1^2\ \ \overline{n}_1^1\ 1\,) = \frac{\overline{n}_1^2(\overline{n}_1^1 - 1)}{\overline{n}_1^1(\overline{n}_1^2 - 1)}\,. \tag{11}$$

Similarly to [4], we can obtain the analogous expression for the change n_1^{21} of the transformation ratio so that the following relationships are performed

$$n_1^{21} = \frac{m_n^{21} - 1}{m_n^{21} + 1}\,,\ m_n^{21} = \frac{n_1^{21} + 1}{1 - n_1^{21}}\,. \tag{12}$$

For this purpose, we make substitution of variables that to use ready expressions [4]. Let us introduce the value

$$\tilde{n}_1 = 2\overline{n}_1 - 1\,. \tag{13}$$

According to [4], the change $\tilde{n}_1^{21}$ has the form

$$\tilde{n}_1^{21} = \frac{\tilde{n}_1^2 - \tilde{n}_1^1}{1 - \tilde{n}_1^2 \tilde{n}_1^1}\,.$$

Using substitution of variables (13), we get

$$\tilde{n}_1^{21} = \frac{\overline{n}_1^2 - \overline{n}_1^1}{\overline{n}_1^2 + \overline{n}_1^1 - 2\overline{n}_1^2 \overline{n}_1^1} = n_1^{21}. \tag{14}$$

In this expression the changes of the variables are equal to among themselves, $\tilde{n}_1^{21} = n_1^{21}$.
Using (14), we obtain the subsequent value n_1^2 of the transformation ratio

$$\overline{n}_1^2 = \frac{\overline{n}_1^1(1 + n_1^{21})}{1 + n_1^{21}(2\overline{n}_1^1 - 1)}\,. \tag{15}$$

There is a group transformation. Moreover, if the initial value $\overline{n}_1^1 = 1$, then the subsequent value $\overline{n}_1^2 = 1$ regardless of the value n_1^{21}. Therefore, movement of a point is realized in hyperbolic geometry [10].

Similarly to the above, let us consider the cross ratio for the load resistance R_1. The cross ratio for the initial point R_1^1, relatively to the base points $0, R_{1\min}$, and a unit point $R_1 = \infty$ has the form

$$m_R^1 = (\,0\ \ R_1^1\ \infty\ R_{1\min}\ ) = \frac{n_1^1}{1 - 4\dfrac{R_i}{R_1^1}\dfrac{(U_{1=})^2}{U_0^{\,2}}}\,. \tag{16}$$

Expression (16) equals the corresponding cross ratio for the conductance $Y_1 = 1/R_1$ and load current $I_1 = U_{1=}/R_1$.
The following equality takes place

$$m_R = (m_n)^2 . \tag{17}$$

This expression leads to identical values if we use the hyperbolic metric to determine the regime value as the distance

$$S = Ln\, m_R = 2Ln\, m_n . \tag{18}$$

Similarly to (10), the cross ratio m_R^{21}, which corresponds to a regime change $R_1^1 \to R_1^2$, has the view

$$m_R^{21} = (\,0\;\; R_1^2\;\; R_1^1\;\; R_{1\min}\,) = \frac{R_1^2\,(R_1^1 - R_{1\min})}{R_1^1\,(R_1^2 - R_{1\min})}. \tag{19}$$

We can introduce the change R_1^{21} of the load resistance by the following expression

$$m_R^{21} = \frac{1 + R_1^{21}}{1 - R_1^{21}} . \tag{20}$$

We use the values $\overline{R}_1^2 = R_1^2 / R_{1\min}$, $\overline{R}_1^1 = R_1^1 / R_{1\min}$.
Similarly to (14), we get

$$R_1^{21} = \frac{\overline{R}_1^2 - \overline{R}_1^1}{\overline{R}_1^2 + \overline{R}_1^1 - 2\overline{R}_1^2\overline{R}_1^1} . \tag{21}$$

Then, ***there is a strong reason to introduce the changes of the load resistance R_1^{21} and the transformation ratio n_1^{21} as expressions (21), (14).***

The validity of such definitions for changes is confirmed by the following expression similar to initial expression (6)

$$R_1^{21} = \frac{2n_1^{21}}{1 + (n_1^{21})^2} . \tag{22}$$

Using (21), we obtain the subsequent value

$$\overline{R}_1^2 = \frac{\overline{R}_1^1(1 + R_1^{21})}{1 + R_1^{21}(2\overline{R}_1^1 - 1)} . \tag{23}$$

As well as (15), if the initial value $\overline{R}_1^1 = 1$, then the subsequent value $\overline{R}_1^2 = 1$ regardless of the value R_1^{21}.

Thus, ***the concrete kind of a circuit and character of regime imposes the requirements to definition of already system parameters.***

Therefore, arbitrary and formal expressions for the regime parameters are excluded.

B. Stabilization of voltage of two loads

Let us consider a circuit with two loads in Fig.1. Variation of one of loads leads to mutual change of stabilization regimes for both loads. Therefore, it is necessary to change the transformation ratios n_1, n_2 in coordination.

Equation (2), taking into account the second load power, becomes as

$$\frac{R_i}{R_1}U_1^2 + \frac{R_i}{R_2}U_2^2 + \left(U - \frac{U_0}{2}\right)^2 = \frac{U_0^2}{4} . \tag{24}$$

By definition,

$$n_1 = U_1 / U\, ,\, n_2 = U_2 / U . \tag{25}$$

Using (24), (6), and (25), we get the first load voltage

$$U_1 = \frac{n_1 U_0}{1 + \frac{R_i}{R_1}n_1^2 + \frac{R_i}{R_2}n_2^2} . \tag{26}$$

It follows that

$$\frac{R_i}{R_1}n_1^2 + \frac{R_i}{R_2}n_2^2 - \frac{n_1 U_0}{U_1} + 1 = 0 . \tag{27}$$

By definition (25)

$$n_2 = n_1 U_2 / U_1 .$$

Substituting this expression in (27), we get

$$n_1{}^2 R_i\left[\frac{1}{R_1} + \frac{1}{R_2}\left(\frac{U_2}{U_1}\right)^2\right] - \frac{n_1 U_0}{U_1} + 1 = 0 . \tag{29}$$

The expression in the square brackets is the total resistance R_T of both loads relatively to the first load. Then

$$n_1{}^2\frac{R_i}{R_T} - \frac{n_1 U_0}{U_1} + 1 = 0 .$$

This expression corresponds to (7) and the dependence $R_T(n_1)$ coincides with Fig.3. Therefore, for the given load resistance, we determine R_T, n_1, and n_2.

Conclusion

Non-Euclidean geometrical interpretation of graphical charts defines the load and regime parameters, and describes the movement of an operating point along the regulation curve. Obtained expressions can be generalized for three or more loads.

References

[1] D. M. Yehia, Y. Yokomizu, D. Iioka, T. Matsumura, "Deliverable-power dependence on distribution-line resistance and number of loads in low-voltage DC distribution system," IEEJ Transactions on Electrical and Electronic Engineering, vol. 7, 2012, pp. 23–30.

[2] X. Lui, P. Wang, and P. Loh, "A Hybrid AC/ DC micro grid and its coordination control ", *IEEE Transactions on Smart Grid*, vol. 2 no. 2, 2011, pp. 278-286.

[3] S. Chae, B. Hyun, P. Agarwal, W. Kim, and B. Cho, "Digital predictive feed-forward controller for a DC–DC converter in plasma display panel," *IEEE Transactions on Power Electronics*, vol. 23, no. 2, 2008, pp. 627-634.

[4] A. Penin, "Non-Euclidean geometrical transformation groups in the electric circuit theory with stabilization and regulation of load voltages", *International journal of circuits, systems and signal processing,* vol. 8, 2014, pp.182-194.
http://www.naun.org/main/NAUN/circuitssystemssignal/2014/a322005-077.pdf

[5] A. Penin, Analysis of electrical circuits with variable load regime parameters. Projective geometry method, Springer International Publishing AG, in press.

[6] "Stereographic projection", Encyclopedia Wikipedia,
http://en.wikipedia.org/wiki/Stereographic_projection

[7] S. Singer and R. Erickson, "Power-source element and its properties,"

[8] D. Maksimovic, Y. Jang, and R. Erickson, "Nonlinear- carrier control for high-power-factor boost rectifiers," IEEE Transactions on power electronics, vol.11, no.4, 1996, pp. 578-584.

[9] J. A. Frank, Schaum's outline of theory and problems of projective geometry, McGraw-Hill, 1967.

[10] "Hyperbolic geometry," Encyclopedia Wikipedia, http://en.wikipedia.org/wiki/Hyperbolic_geometry