Voltage Regulation and Stabilization with Limited Capacity Power Supply

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***Abstract* — The restriction of load power, two–valued regulation characteristic, and interference of loads is observed in power supply systems with limited power of voltage source. The interpretation of load regime changes is presented by using the hyperbolic plane. This geometrical interpretation allows basing the definition of operating regime parameters. Results can be useful for electric circuit theory education and for voltage coordinated control of given loads.**
***Index Terms* — cross ratio, limited voltage source, projective transformations, regulated characteristics.**

I. INTRODUCTION

In power supply systems with limited voltage source power, the restriction of load power, two–valued regulation characteristic, and interference of several loads is observed [1]. Distributed power supply systems, autonomous or hybrid power supply systems on the basis of solar cells, fuel elements, and accumulators can be examples of such systems [2]. At present time, a feed– forward control method is used [3]. This method calculates the required duty ratio variation by the predicted load current. Therefore, it is necessary to take into account the internal resistance of power supply, to carry out analysis of the load interference and obtain relationships for definition of regime parameters.

The approach for interpretation of changes or "kinematics" of load regimes is presented by using the conformal and hyperbolic plane [4]. In the present paper, the base of this approach is interpreted.

II. BASIC MODEL OF VOLTAGE REGULATORS. DISPLAY OF CONFORMAL GEOMETRY

We consider a power supply system with two idealized voltage regulators $VR1, VR2$, and loads R_1, R_2 in Fig.1. The regulators define transformation ratios

$$n_1 = \frac{V_1}{V}, \; n_2 = \frac{V_2}{V}. \qquad (1)$$

An interference of the regulators on load voltages V_1, V_2 is observed by an internal resistance R_i . The equation of this circuit at change of parameters n_1, n_2 has the view

$$\frac{R_i}{R_1}(V_1)^2 + \frac{R_i}{R_1}(V_2)^2 + \left(V - \frac{V_0}{2}\right)^2 = \frac{(V_0)^2}{4}. \qquad (2)$$

Let the first load voltage $V_{1=}$ be stabilized, $V_2 = \text{var}$, and $R_2 = const$. But for all that, the first load resistance may be both positive $R_1 > 0$ and negative $R_1 < 0$. Also, the second load resistance is positive $R_2 > 0$.

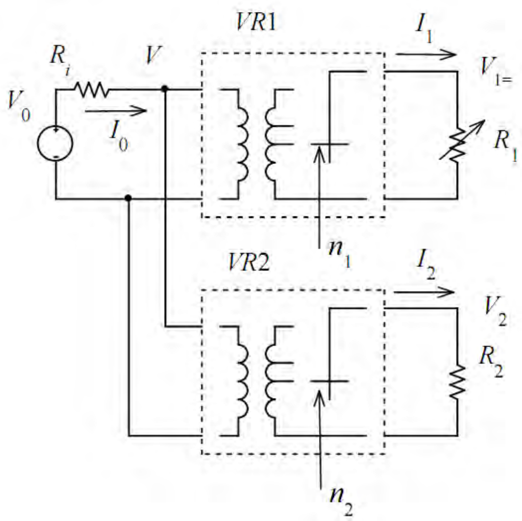

Fig.1 Power supply system with two regulators and loads

For example, the circuit in Fig.1 corresponds to the positive load $R_1 > 0$ and pulse regulators in Fig.2 conform to the negative load $R_1 < 0$. Expression (2) corresponds to a surface with a parameter R_1 in the coordinates V_1, V_2, V . If $R_1 > 0$, this expression represents a sphere (ellipsoid). The both loads consume energy; the voltage source V_0 gives energy. If $R_1 < 0$, we get a one-sheeted hyperboloid. The first load, as a constant voltage source $V_{1=}$, gives energy. In addition, the voltage source V_0 , as energy storage, may consume and give back energy. The corresponding direction of the current I_0 determines these regimes. If $R_1 = \infty$, as the open circuit regime, the corresponding surface degenerates into a cylinder.

For realization of the given regime, it is necessary to change the transformation ratios n_1, n_2 in some coordination. Let us obtain the required relationships.

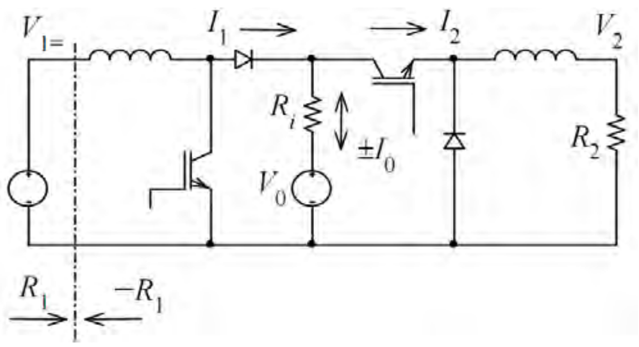

Fig.2 Power supply system with negative resistance $-R_1$

So, the plane with the parameter $V_{1=}$ intersects the bunch of spheres and hyperboloids. As the result of this section, the bunch of circles in coordinates V_2, V is obtained, as it is shown in Fig.3(a).

In this case, our expression has the form

$$\frac{R_i}{R_2}(V_2)^2 + \left(V - \frac{V_0}{2}\right)^2 = \frac{(V_0)^2}{4} - \frac{R_i}{R_1}(V_{1=})^2 . \qquad (3)$$

The second member of this equation is the radius of a circle for the given value R_1. It is possible to consider the voltage V_2 change as the radius-vector rotation. This rotation determines a point movement along the straight line $V_{1=}$ in coordinates V_1, V in Fig.3(b).

In the general case of the variable V_2, we get the surfaces, which rotate around the diameter $V = V_0 / 2$, as it is shown by closed arrows. Also, the transformation ratios n_1, n_2 are resulted by the stereographic projection of sphere's points on the tangent plane or conformal plane. The axes n_1, n_2 are superposed in Fig.3(b). Using (1), we obtain the equation

$$\frac{R_i}{R_1}(n_1)^2 + \frac{R_i}{R_2}(n_2)^2 - \frac{n_1 V_0}{V_{1=}} + 1 = 0 . \qquad (4)$$

This equation circumscribes trajectories for different values R_1, as it is shown in Fig.4. These trajectories are characteristic lines for the conformal plane. If $R_1 > 0$, the bunch of circles with parameters R_1^1, R_1^2 is obtained. For $R_1 = \infty$, equation (4) corresponds to a parabola. The case $R_1 < 0$ conforms to a hyperbola. For the limit values $R_1 = 0$ and $n_1 = 0$, the hyperbola degenerates and coincides with the axis n_2.

The radius-vector rotation in the plane $V_2\ V$ determines the analogous rotation in the plane $n_1\ n_2$ with the centre $n_{1\max}$.

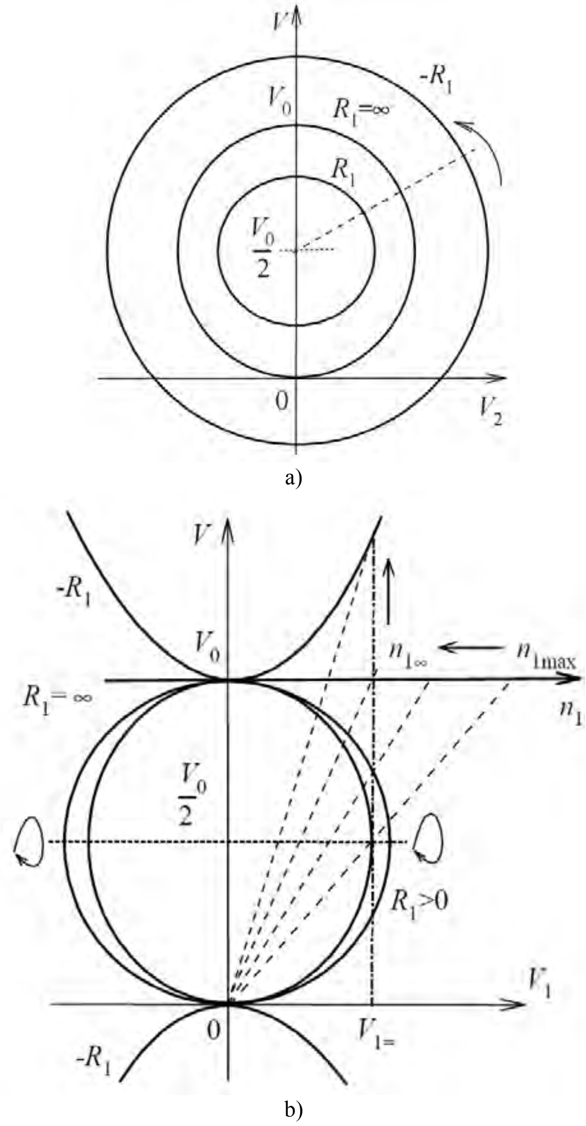

Fig.3 Bunch of circles for different R_1 as $V_{1=}$ (a) and bunch of curves at $V_2 = 0$ and a point moving along lines $V_{1=}$ (b)

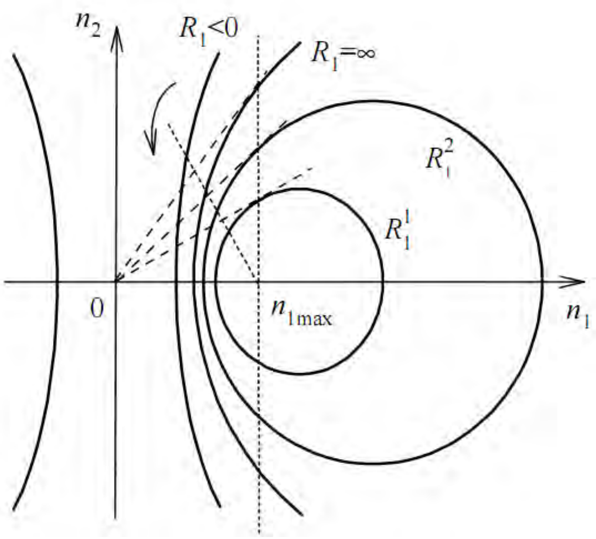

Fig.4 Trajectories for different values R_1 as $V_{1=}$

This center corresponds to the minimum load resistance $R_{1\min}$, where

$$R_{1\min} = 4R_i \frac{(V_{1=})^2}{(V_0)^2}, \quad n_{1\max} = 2\frac{V_{1=}}{V_0}. \quad (5)$$

In turn, the voltage V_2 is directly proportional to the voltage $V_{1=}$, $V_2 = V_{1=} n_2 / n_1$. That is, we have a straight line, which intersects the bunch of circles with the parameter R_1 in two points. Then, the tangent lines to the circles (curves) determine the points of the maximum voltage $V_{2\max}$, the voltage $V = V_0 / 2$. In this case,

$$V_{2\max} = \frac{V_0}{2}\sqrt{\frac{R_2}{R_i}}\sqrt{1-\frac{R_{1\min}}{R_1}}, \quad n_{2\max} = \frac{V_0}{2} V_{2\max}. \quad (6)$$

On the one hand, expression (4) corresponds to the regulation rule. But, on the other hand, the operating area of the transformation ratio is limited by the value $n_1 \le n_{1\max}$. So, we must decrease the next value n_1, for the next regulator switching period, by some rule. In this sense, we come to hyperbolic geometry.

III. USE OF HYPERBOLIC GEOMETRY

We may suppose that the straight line $n_{1\max}$ in the plane $n_2\ n_1$ is the infinitely remote line. Therefore, geometry of the half plane $n_2, n_1 \le n_{1\max}$ in Fig.4 and of normalized half plane $\overline{n}_2\ \overline{n}_1 \le 1$ in Fig.5(a) corresponds to Poincare's model of hyperbolic geometry. This Poincare's model is also demonstrated by projective coordinates g_2, g_1 for the left-hand figure and by these Cartesian coordinates for the right-hand figure in Fig.5(b) for the half plane $g_2, g_1 \ge 0$. For Poincare's model of hyperbolic geometry, the half-rounds with the given resistance R_1 intersect the axes g_2 orthogonally. Let us introduce this geometry. To do this, it is necessary to change the variables $n_2(g_1, g_2),\ n_1(g_1, g_2)$ so that all curves of the plane $n_2\ n_1$ would be converted into circles of the plane $g_2\ g_1$. Further, we use the normalized values

$$\overline{n}_1 = \frac{n_1}{n_{1\max}}, \quad \overline{n}_2 = \frac{n_2}{n_{2ref}}.$$

As the scale value n_{2ref}, we may use a circle with some characteristic value of the parameter R_1. The resistance value $R_1 = \infty$ may be such a characteristic value. Then

$$n_{2ref} = \sqrt{\frac{R_2}{R_i}}. \quad (7)$$

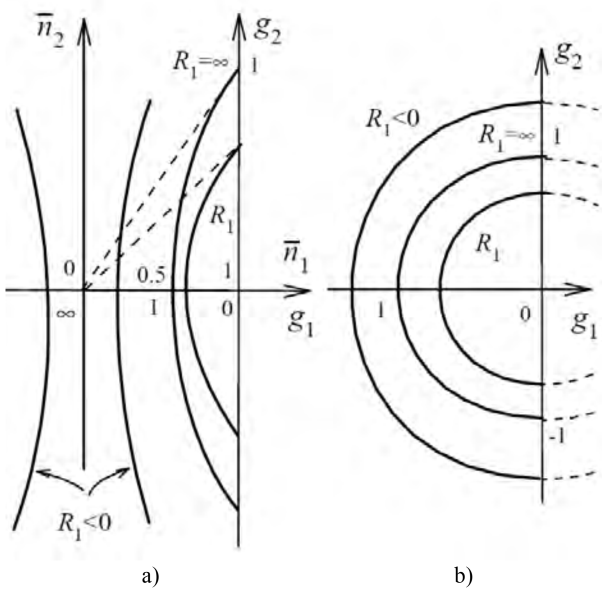

Fig.5 Poincare's model of hyperbolic geometry for superposed half planes $\overline{n}_2, \overline{n}_1 \le 1$ and $g_2, g_1 \ge 0$ (a) and half plane for orthogonal coordinates g_2, g_1 (b)

It is possible to represent expression (4) in the normalized form

$$\frac{R_{1\min}}{R_1}(\overline{n}_1)^2 + (\overline{n}_2)^2 - 2\overline{n}_1 + 1 = 0. \quad (8)$$

The required change of variables has the view

$$\overline{n}_1 = \frac{1}{1+g_1}, \quad \overline{n}_2 = \frac{g_2}{1+g_1}. \quad (9)$$

In this case, equation (8) transforms into the equation of circle

$$(g_1)^2 + (g_2)^2 = 1 - \frac{R_{1\min}}{R_1} = (r_1)^2. \quad (10)$$

The second member of this equation is a radius squared for the given R_1. We may term the variables g_1, g_2 as hyperbolic transformation ratios.

This geometric model allows to use a cross ratio for the determination of regimes and their change.

IV. REGIME CHANGE FOR THE FIRST GIVEN LOAD RESISTANCE

For clarity, let us consider the half-rounds with the parameter R_1^1 in Fig.6. Let points C_{1g}, D_{1g} be points of an initial and subsequent regime.

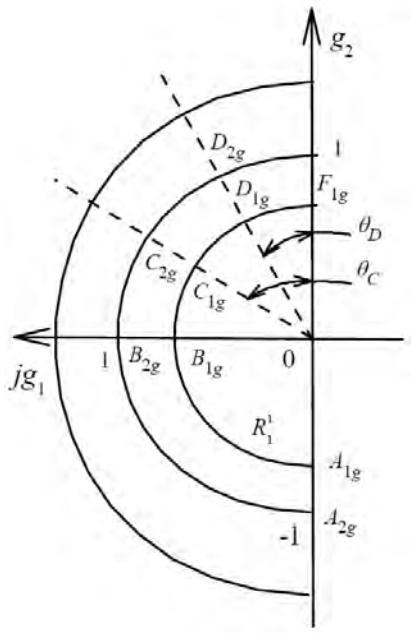

Fig.6 Regime change for Poincare's model of hyperbolic geometry

Then, the cross ratio, which corresponds to the regime change $C_{1g} \to D_{1g}$, has the form

$$m_g^{DC} = \frac{D_{1g} - A_{1g}}{D_{1g} - F_{1g}} \div \frac{C_{1g} - A_{1g}}{C_{1g} - F_{1g}}.$$

The coordinates of all the points $A_{1g}, D_{1g}, C_{1g}, F_{1g}$ are given by complex numbers as follows

$$g^{A1} = g_2^{A1} + j0,\ g^{D1} = g_2^{D1} + jg_1^{D1},$$

$$g^{C1} = g_2^{C1} + jg_1^{C1},\ g^{F1} = g_2^{F1} + j0.$$

In particular, radius of half-rounds (10) defines the coordinates g_2^{A1}, g_2^{F1} as the follows

$$g_2^{A1} = -r_1,\ g_2^{F1} = r_1.$$

For Poincare's model of hyperbolic geometry by Fig.6, cross ratio (11) looks like

$$m_g^{DC} = \frac{tg\theta_C}{tg\theta_D} = \frac{r_1 - g_2^{C1}}{g_1^{C1}} \div \frac{r_1 - g_2^{D1}}{g_1^{D1}}.$$

Using (10), we get

$$\left(m_g^{DC}\right)^2 = \frac{r_1 - g_2^{C1}}{r_1 + g_2^{C1}} \div \frac{r_1 - g_2^{D1}}{r_1 + g_2^{D1}}.$$

This expression gives the subsequent value

$$\frac{g_2^{D1}}{r_1} = \frac{\dfrac{g_2^{C1}}{r_1} + g_2^{DC}}{1 + \dfrac{g_2^{C1}}{r_1} g_2^{DC}},$$

where the hyperbolic transformation ratio change is introduced as

$$g_2^{DC} = \frac{\left(m_g^{DC}\right)^2 - 1}{\left(m_g^{DC}\right)^2 + 1}.$$

This change corresponds to the points C_{2g}, D_{2g} of the half-rounds with parameter R_1^2 and so on.

We must obtain an expression for the subsequent value of the transformation ratios n_1, n_2. Using (9), we calculate the subsequent value of transformation ratios n_1, n_2.

V. CONCLUSION

Geometrical interpretation allows basing the definition of operating regime parameters. The concrete kind of circuit and character of regime determines system parameters; arbitrary expressions are excluded.

Obtained expressions can be generalized for other kind of regimes, circuits and number of loads.

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