On some determination solutions of the stationary Navier-Stokes equation

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The following system of partial differential equations are examined:

$$\begin{cases} \frac{P_x}{\mu} + uu_x + vu_y = a(u_{xx} + u_{yy}) + F_x \\ \frac{P_y}{\mu} + uv_x + vv_y = a(v_{xx} + v_{yy}) + F_y \\ u_x + v_y = 0 \end{cases} \tag{1}$$

$$P = P(x, y); \quad u = u(x, y); \quad v = v(x, y); \quad x, y \in \mathbb{R}.$$

The system (1) describes the process stationary flow of a liquid or gas on a plane surface. The P function represent the pressure of the liquid, and u, v functions represent the flow of the liquid (gas). The constants $a > 0$ and $\mu > 0$ is a determined parameter of the liquid's (of the gas) viscosity and liquid's density. $F = (F_x, F_y)$ represent the exterior forces.

The following theorem is proved: If in the area D u and v functions admits the derivatives bounded up to including order 2 and the function F ; if z is analyticaly in the aria D function, we are to determine the solution of system (1):

$$u = Im\, z, \quad v = Re\, z, \quad P = F - 0,5(u^2 + v^2) + C.$$

The analyzed cases if u and v functions is not $u = Im\, z, v = Re\, z$, if z is analyticaly function, and is to determine different particular and exact solutions of the system (1).

References

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