On integrability of homogeneous fractional quadratic differential equation

COZMA Dumitru and MATEI Angela

We consider the homogeneous fractional quadratic differential equation of the form

$$\frac{dy}{dx} = \frac{b_{20}x^2 + b_{11}xy + b_{02}y^2}{a_{20}x^2 + a_{11}xy + a_{02}y^2}, \tag{1}$$

where a_{ij}, b_{ij} are real parameters and the right hand side of (1) is irreducible.

The problem of the existence of polynomial inverse integrating factors and non-existence of limit cycles in equation (1) was investigated in [1] and the problem of construction of integral curves in homogeneous equations (1) was studied in [2]. We study the integrability of equation (1) with algebraic solutions and prove that the equation (1) always has at least one invariant straight line of the form $y = kx$, where k is the solution of the equation

$$a_{02}k^3 + (a_{11} - b_{02})k^2 + (a_{20} - b_{11})k - b_{20} = 0.$$

We show that when $a_{20} \neq b_{11}$, the equation (1) has an integrating factor of the form

$$\mu = \frac{1}{b_{20}x^3 + (b_{11} - a_{20})x^2y + (b_{02} - a_{11})xy^2 - a_{02}y^3}. \tag{2}$$

Based on the integrating factor (2) we construct the first integrals in all cases of the differential equation (1).

References:

1. A.M. Hussien. Polynomial inverse integrating factors, first integral and non-existence of limit cycles in the plane for quadratic systems. Science Journal of University of Zakho, 2017, vol. 5, no. 2, p. 232–238.

2. Mironov A.N., Sozontova E.A., On construction of integral curves of homogeneous equations. Elabuga, 2009.

(COZMA Dumitru, MATEI Angela) Tiraspol State University, Chişinău, Republic of Moldova
E-mail address: dcozma@gmail.com