Four-dimensional reductive Lie algebra for the ternary differential system with quadratic nonlinearities and its perspectives in the study of this system

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Consider the ternary differential system

$$\frac{dx^j}{dt} = a^j_\alpha x^\alpha + a^j_{\alpha\beta} x^\alpha x^\beta \ (j, \alpha, \beta = 1, 2, 3), \tag{1}$$

where the tensor coefficient $a^j_{\alpha\beta}$ is symmetric in the lower indices, by which a total convolution is carried out here. The coefficients and variables of this system take values from the fields of real numbers $\mathbb{R}$.

In the study of system (1) in Chişinău, a reductive Lie algebra L_9 was used to represent the centro-afine group $GL(3, \mathbb{R})$ in the space of the coefficients of this system.

In the doctoral theses Natalia Gerştega [1, 2006], Oxana Diaconescu (Cerba) [2, 2008] and Natalia Neagu [3, 2018] were studied using invariants and comitants these algebras, the dimension of $GL(3, \mathbb{R})$-orbites, $GL(3, \mathbb{R})$-invariant integrals and stability by Lyapunov of unperturbed motion governed by the system (1). But the large dimension of algebra L_9 does not allow us to make extensive use of these studies of mentioned system. That is why the question arose, if it is not a smaller dimension reductive Lie algebra, what is a subalgebra of algebra L_9, with which we could more easily study the system (1).

In [1] were determined such Lie algebras of dimension 4, isomorphic with Lie algebra corresponding to the linear representation of the centro-affine group $GL(2, \mathbb{R})$ in the space of the coefficients of two-dimensional differential polynomial systems [4, 5], but which proved to be ineffective.

In this paper it was shown that there is also a reductive subalgebra L_4 of the algebra $L(9)$, which is not isomorphic with the above mentioned algebra, but which is closely related to the rotation and extension groups, which are groups with high geometric value.

Using Lie algebra L_4 was determined the functional basis of comitants and invariants of system (1) in relation to the rotation and extension groups, which contains the basis of these invariant polynomials in relation to the centro-affine group $GL(3,\mathbb{R})$.

References

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