The Iterative Algorithm of Tuning Controllers to the Models Object with Inertia and Astatism

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Abstract **— This paper proposes an iterative algorithm of tuning the typical controllers P, PI, PID to the model objects with inertia second order and astatism. The proposed algorithm is using the maximal stability degree method for tuning controllers. As the result of this study the algorithm of tuning controllers and the procedure of determining the system's performance in dependence of maximal stability value proposed.**

Index Terms **— the maximal stability degree method, the iterative algorithm, tuning of controllers**.

I. INTRODUCTION

During the automation of many slow technological processes the mathematical objects' models of control process are represented as the models with respectively order inertia and astatism [1, 2].

The procedure of tuning controllers to the model object with inertia and astatism becomes difficult [1,4,5]. This paper analyses the model object (fixed part) with inertia (second order) and astatism with transfer function, which is presented in the follow form

$$H_{PF}(s)=\frac{k}{s(T_1s+1)(T_2s+1)}=\frac{k}{a_0s^3+a_1s^2+a_2s}, \qquad (1)$$

where $k, T_1, T_2, a_0, a_1, a_2$ are the parameters of object.

At the projecting of control system are used many tuning methods of typical regulators: Ziegler-Nichols method, frequency method, criteria (of modulus) method etc. Frequency method is accompanied with difficulties of calculating [1-5]. Criteria (of modulus) method becomes unacceptable when the control processes are slowly, and they have big time constants and this reduce the performances of entire system. Because the model of object (1) is complex, some of known tuning methods of regulators for these objects either can't be used (the Ziegler-Nichols method) or they are accompanied by difficult calculations (the parametrical optimization method).

It is proposed to tune the standard controllers P, PI, PID using the maximal stability degree (M.S.D.) method [3, 6-9] to the model object (1) with known parameters k, T_1, T_2, a_0, a_1, a_2 and to analyse the dynamic of control system for the case when it varies the object's parameters from the nominal values k, T_1, T_2 keeping the tuning parameters of controllers P, PI and PID.

II. THE ITERATIVE ALGORITHM OF TUNING CONTROLLERS

We assume that the control system is formed of an object with transfer function $H_{PF}(s)$, which is presented in relation (1), and transfer function of controller $H_R(s)$ with typical control laws P, PI, PID, Fig. 1.

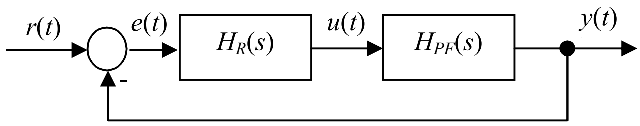


Fig.1. Structure scheme of control system.

It will tune the typical algorithms of tuning P, PI and PID for the model object with known parameters, using the M.S.D. method (iterative algorithm).

For the tuning of the P controller using the M.S.D. method the algebraic expressions [6-9] were applied

$$-a_0J^2+a_1J+a_2J=0, \qquad (2)$$

$$k_p=(J/k)(a_0J^2-a_1J+a_2). \qquad (3)$$

The optimal values of parameters k_p of P controller were determined from relation (3).

For the tuning the PI controller using the M.S.D. method the algebraic expressions [4, 5] were applied

$$6a_0J^2-3a_1J+a_2=0, \qquad (4)$$

$$k_p=(J/k)(4a_0J^2-3a_1J+2a_2), \qquad (5)$$

$$k_i=1/T_i=(1/k)(-a_0J^4+a_1J^3-a_2J^2)+k_pJ. \qquad (6)$$

The optimal values of parameters k_p and k_i of PI controller were determined from relations (5) and (6).

In the case of tuning parameters of PID controller using the M.S.D. method the algebraic expressions [4, 5] were applied

$$4a_0J-a_1=0, \qquad (7)$$

$$k_p=(J/k)(4a_0J^3-3a_1J^2+2a_2J)+2k_dJ, \qquad (8)$$

$$k_i=1/T_i=(1/k)(-3a_0J^4+a_1J^3-a_2J^2)-k_dJ^2+k_RJ, \qquad (9)$$

$$k_d=(1/k)(-6a_0J^2+3a_1J-a_2), \qquad (10)$$

From expressions (8)-(10) were determined the optimal values of parameters k_p, k_i and k_d of PID controller.

The expressions (2), (4), (7) is a function of object's parameters and unknown value of stability degree J. Solved the expressions (2), (4), (7) it is determined the optimal value of stability degree J_{opt}, which presented the smallest positive and real root or the real positive part of complex root.

The optimal values of parameters k_p, k_i and k_d of P, PI and PID controllers were determined from relations (3), (5), (6), (8), (9) and (10) for each type of controller, but for this optimal values of controllers parameters it can not be obtained the optimal performance of the control system.

To overcome this difficulty we propose the following iterative algorithm of tuning P, PI and PID controllers to the model object with known parameters.

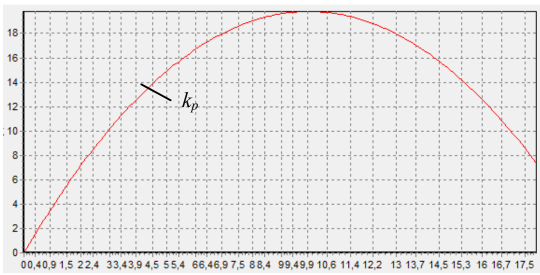
The tuning parameters of P, PI and PID controller - k_p, k_i and k_d are the function of known parameters of control object and of the unknown value J stability degree of control system: k_p=f(J), k_i=f(J), k_d=f(J) (see relations (3), (5), (6), (8), (9) and (10)). Based on these relations in the case of known object's parameters and in the case of variation stability degree $J \geq 0$ in the strict limits, the respective calculations were made and the dependences k_p=f(J) for P controller, k_i=f(J) and k_p=f(J) for PI controller, k_i=f(J), k_p=f(J) and k_d=f(J) for PID controller were obtained.

A simulation was made to verify the performance of control system. If the performance doesn't sutisfy the imposed performance, the iterative process will choose other sets of values of controllers parameters from curves k_p=f(J), k_i=f(J), k_d=f(J), the procedure will repeat until the performance of system will be satisfied.

III. APPLICATION AND COMPUTER SIMULATION

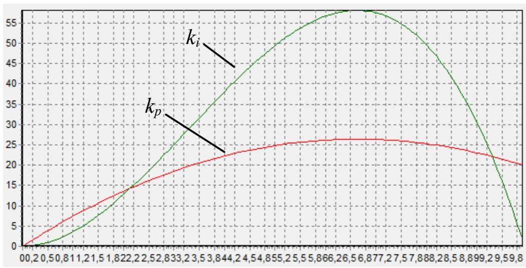
To show the efficiency of the proposed iterative algorithm for tuning the typical controllers P, PI, PID to the model of object (1), which has the following parameters k=0.25, T_1=0.001 s, T_2=0.05 s and a_0=0.00005, a_1=0.051, a_2=1 the procedure described above for tuning typical controllers will be used. It was proposed the variation of constant time T_2 with 100 %.

For the realtions k_p=f(J), k_i=f(J), k_d=f(J) of P, PI and PID controllers (see realtions (3), (5), (6), (8) - (10)) with the known parameters values of object and at the variation of the stability degree J in the respectively limits the iterative calculations were made and the obtained results are presented in the Fig. 2, 3, 4.



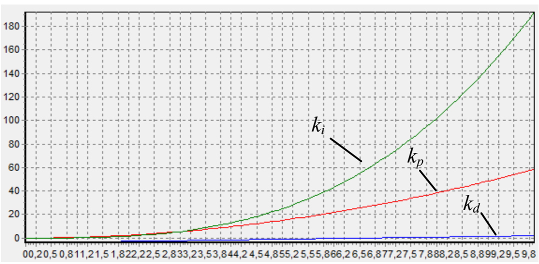
The value of stability degree

Fig. 2. The dependence of P controller parameter of the stability degree value.



The value of stability degree

Fig. 3. The dependence of PI controller parameters of the stability degree value.



The value of stability degree

Fig. 4. The dependence of PID controller parameters of the stability degree value.

To verify the obtained results in case of tuning controllers P, PI, PID to the model object (1) using the M.D.S. method a computer simulation of the control system in MATLAB was made (Fig. 5).

For analyze the set of performance of control system with P, PI and PID controllers from Fig. 2, 3 and 4 it was chosen the sets of values J - k_p for the P controller (Table I), J - k_p and k_i for the PI controller (Table II) and J - k_p, k_i, k_d for the PID controller (Table III) and it was made the computer simulation of the control system with P, PI and PID controller in MATLAB was made (Fig. 5) and the obtained transition processes are presented in the Fig. 6, 7, 8, where the number of the curves it is coincided with the number of the values of the set's variant from Tables I, II and III (the marked position 3 from Table I, position 3 from Table II and the position 2 from Table III present the optimal values). The line 7 from Tables I, II and III represent the values obtained for the procedure of optimization from MATLAB. The Ziegler-Nichols method is not applicable for the model object presented with relations (1).

TABEL I. THE VALUES OF THE P CONTROLLER'S PARAMETERS

Item	J	k_p
1	5	14,92
2	8	19,04
3	**10**	**19,8**
4	13	17,96
5	17	10
6	10	19,8
7		23,5719

TABEL II. THE VALUES OF THE PI CONTROLLER'S PARAMETERS

Item	J	k_p	k_i
1	2,1	14,11	13,87
2	4	22,26	38.04
3	**6,6**	**26,37**	**58,08**
4	7	26,29	57,5
5	7,5	25,91	54,77
6	7,5	25,91	54,77
7		74,0846	53,3876

TABEL III. THE VALUES OF THE PID CONTROLLER'S PARAMETERS

Item	J	k_p	k_i	k_d
1	7	28,61	65,66	0,17
2	**8**	**37,41**	**98,24**	**0,76**
3	9	47,35	140,05	1,35
4	10	58,43	192,18	1,94
5	10,9	69,37	248,82	2,47
6	10,9	69,37	248,82	2,47
7		109,3078	0,3644	0,9758

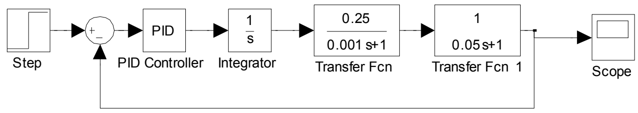


Fig. 5. Simulation diagrams of the control system.

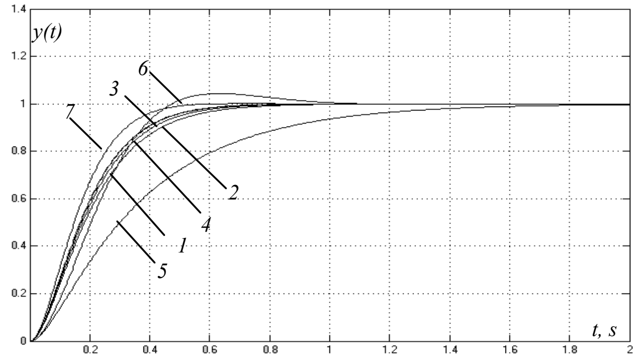


Fig. 6. The transient processes of control system with P controller.

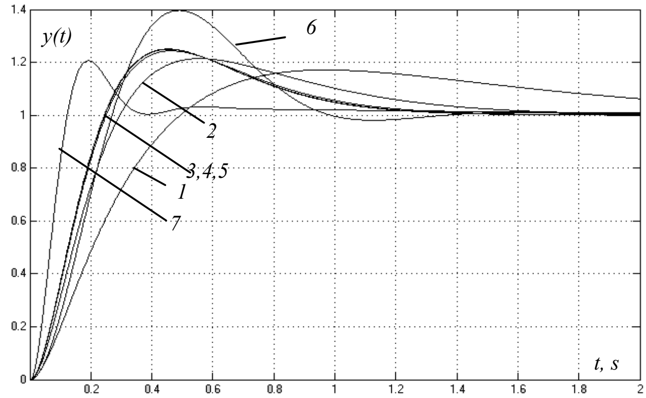


Fig. 7. The transient processes of control system with PI controller.

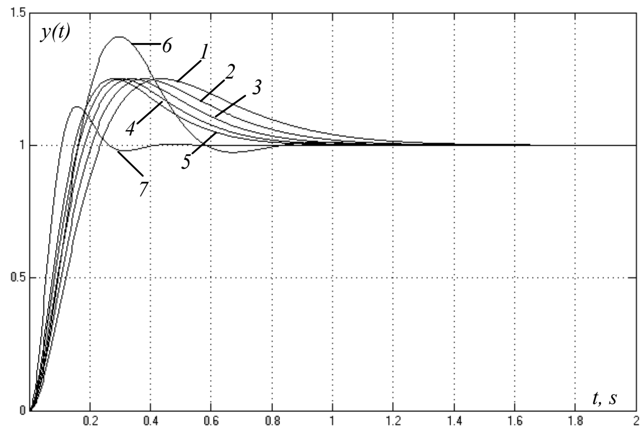


Fig. 8. The transient processes of control system with PID controller.

Analyzing the performance of the control systems from Fig. 6, 7 and 8 it was observed that **the optimal performances** were obtained: for control system with P controller - curve 3 (the marked positions 3 from Tables I and IV), for control system with PI controller - curve 3 (the marked positions 3 from Tables II and V), for control system with PID controller - curve 2 (the marked positions 2 from tables III and VI).

TABEL IV. THE VALUES OF THE P CONTROLLER'S PARAMETERS

Item	t_r, s	σ ,%	λ	t_c, s
1	0,66			0,66
2	0,48			0,48
3	**0,45**			**0,45**
4	0,52			0,52
5	1,06			1,06
6	0,64	5	1	0,41
7	0,48			0,48

TABEL V. THE VALUES OF THE PI CONTROLLER'S PARAMETERS

Item	t_r, s	σ ,%	λ	t_c, s
1	2,14	17,39	1	0,43
2	1,24	22,27	1	0,26
3	**0,97**	**26,28**	**1**	**0,22**
4	0,98	26,15	1	0,22
5	0,97	26,92	1	0,22
6	0,88	40,87	1	0,22
7	0,6	20	1	0,1

In the figure 9, it is presented the comparison of transient processes obtained for optimal values of P, PI and PID controllers where curve 1 – it is presented the control system with P controller, 2 – control system with PI controller and 3- control system with PID controller.

TABEL VI. THE VALUES OF THE PID CONTROLLER'S PARAMETERS

Item	t_r, s	σ ,%	λ	t_c, s
1	0,93	25,15	1	0.21
2	**0,78**	**23,05**	**1**	**0.16**
3	0,65	26,97	1	0.13
4	0,39	32,55	1	0.11
5	0,33	36,74	1	0.09
6	0,84	58,0	3	0,12
7	0,38	14	2	0,1

For the control system with P, PI and PID controller tuning by M.S.D. method which the transition processes are presented in the Fig. 9, were analyzed the distribution of poles of characteristic equations of the control system with P, PI and PID in the complex plan which were calculated in the MATLAB and presented in the Fig. 10.

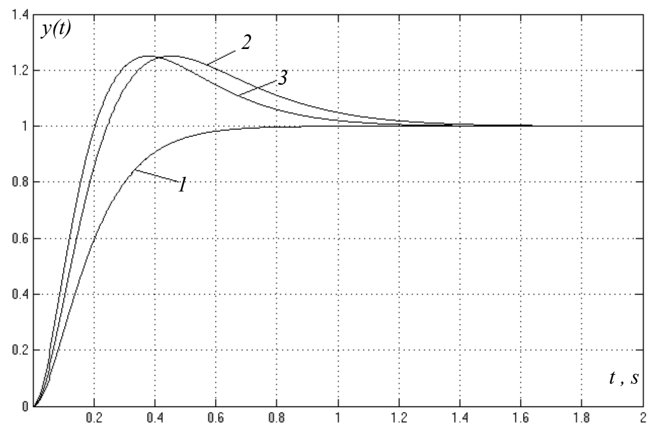


Fig. 9. The transient processes of control systems with P, PI, PID controllers.

For the control system with P, PI and PID controller tuning by MSD which the transition process are presented in the Fig. 9, were analyzed the distribution of poles of characteristic equations of the control system with P, PI and

PID in the complex plan which were calculated in the MATLAB and presented in the Fig. 10.

In the Fig. 10 are presented the domination poles with notations: 1 for the control system with P controller has value - 9,9; 2 for the control system with PI controller has value - 6,44; 3 for the control system with PID controller has value -3,97.

The time constant T_2 of the model object (1) was increased with 100% (from 0.05 to 0.1) and was used the optimal values of the parameters of regulators P, PI and PID indicated in the Tables I, II, III the line 6. The control system with P, PI and PID controller was simulated in MATLAB and the obtained transient processes of the system are presented in Fig. 6, 7, 8 curves 6, the performances of the control system are presented in the Tables IV, V, VI the 6 line. Analyzing the performance of the control system it can be observed that control system kept the quality of functionary.

Analyzing the performance of control system with P, PI and PID controllers it can be observed that the best performance was obtained: after the rise time t_c and overshoot for the control system with PID controller and after control time t_r the best performance was obtained for the control system with PID controller. In general, the priority based on the obtained performance has the control system with PID controller.

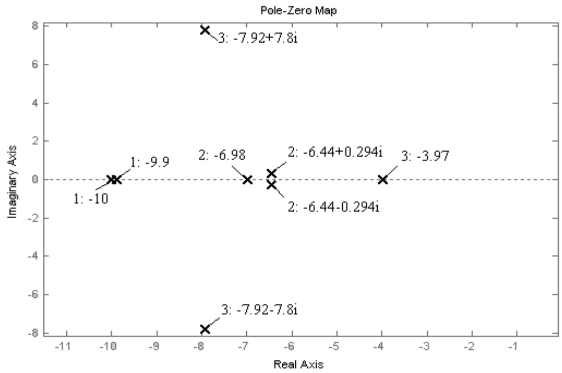


Fig. 10. The distribution of the characteristic equation's poles of the control system with P, PI, PID controller.

The control system with P controller has the stability reserve with two times higher then control system with PI controller and with three times higher then control system with PID controller.

Analyzing the performance of control system and distribution poles of characteristic equations of control system with P, PI and PID controller it can be observed that control system has the guaranteed stability reserve and high robustness.

IV. CONCLUSION

As a result of the study, the following conclusion can be made:

- It is proposed the iterative algorithm (graph-analytical method) of tuning P, PI and PID controllers to the model objects with second order and astatism, which permitted to obtain the settled performance.
- Analyzing the transition processes of the control system with P, PI and PID controller from Fig. 6, 7, 8 it can be seen that making the variation of the degree stability of the control system were obtained the sated performance of the control system (curve 1, 2, 3, Fig. 9). In this case it is necessary to make the iterative calculations for the difference value sets of the *J*- k_p, k_i, k_d for the P, PI and PID controller.
- For the control system with P controller tuning by MSD method, the transition process of system is aperiodical and optimal for the given values of object (view the curve 1, Fig. 9).
- For the control system with PI controller tuning by MSD method, the transition process of system is aperiodic and optimal for the given values of object (see the curve 2, Fig. 9).
- For the control system with PID controller tuning by MSD method, the transition process of system is aperiodic and optimal for the given values of object (see the curve 3, Fig. 9).
- The best performance was obtained for the control system with PID controller.
- Control system with P controller has the stability reserve with two times higher then control system with PI controller and with three times higher then control system with PID controller.
- Tuning controllers P, PI and PID at the given values of the control object (1) using parametrical optimization from MATLAB, the obtained performances of control system are better than the performances of control system with controllers tuning use the maximal stability degree method. In this case the control system lose the robustness (the pole value of control system with PID controller is -0,00333 in comparison with pole value of control system with PID controller using the MSD method, that has value -3,97).
- Varying the constant time T_2 with 100% it was observed that control system operate with high performance.

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