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ON THE LIFETIME AS THE MAXIMUM OR MINIMUM OF THE SAMPLE WITH POWER SERIES DISTRIBUTED SIZE

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Abstract The present paper treats the topic of the distribution of the maximum and minimum of a sequence of an independent identically distributed random variables (i.i.d.r.v.) in a random number in a unitary manner and from the perspective of the power series distribution class. The general formulas are obtained, and by using concrete examples, they lead to some of the distributions obtained by Adamidis and Loukas (1998), Kus (2007), Tahmasbi and Rezaei (2008), Leahu and Lupu (2010), Baretto-Souza, Morais and Cordeiro (2011), Morais and Baretto-Souza (2011), Cancho, Louzada and Barriga (2011), Louzada, Roman and Cancho (2011), Flores, Borges, Cancho and Louzada (2013). Examples of new distributions are also presented.

Keywords: power series distribution, distribution of the maximum, distribution of the minimum.
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1. INTRODUCTION

The introduction of this new (generalized) distribution is connected with reliability problems when lifetime can be expressed as the maximum or minimum of a sequence of i.i.d.r.v., which represents the risk times of the system components. In recent years, some researchers have proposed a series of new distributions for the maximum and minimum of a sequence of i.i.d.r.v.. For example, Adamidis and Loukas [1], Kus [6], Tahmasbi and Rezaei [12], Leahu and Lupu [7], Louzada, Roman and Cancho [9], as well as the Cancho, Louzada and Barriga [3], have been concerned with determining the maximum or minimum distribution when the components in a sequence of i.i.d.r.v. are exponentially distributed, and the number of the components are of a discrete type. Next, Flores, Borges, Cancho and Louzada [4] treat the distribution of a vector's maximum with components that are exponentially distributed in an random number of a power series distribution type. This type of distribution is called complementary exponential power series (CEPS) distribution. Also, Morais and Baretto-Souza [11] considered the analysed Weibull distribution class by means of the power series distribution class (WPS). Recently, Louzada, Bereta and Franco [10] have formulated a mathematical model that unifies the procedure for obtaining a distribution of the maximum and minimum of a sequence of i.i.d.r.v. of the absolutely continuous type in a random number N characterized by the generating function. But the problem of determining the general formula when the r.v. N forms part of a power series distributions remains unsolved.

2. THE POWER SERIES DISTRIBUTION OF THE MAXIMUM AND MINIMUM

Let us consider r.v. Z such that $\mathbb{P}(Z \in \{1, 2, \ldots\}) = 1$.

Definition 2.1. ([5]) *We say that r.v. Z has a power series distribution if:*

$$\mathbb{P}(Z = z) = \frac{a_z \Theta^z}{A(\Theta)}, \ z = 1, 2, \ldots; \ \Theta \in (0, \tau); \tag{1}$$

where $a_1, a_2, \ldots$ are nonnegative real numbers, τ is a positive number bounded by the convergence radius of power series (series function) $A(\Theta) = \sum_{z \geq 1} a_z \Theta^z, \ \forall \ \Theta \in (0, \tau)$, and Θ is power parameter of the distribution (Table 1).

PSD denotes the power series distribution functions class. If the r.v. Z has the distribution from relationship (1), then we write that $Z \in PSD$.

Table 1: The representative elements of the PSD class for various truncated distributions

Distribution	a_z	Θ	$A(\Theta)$	τ
$Binom^*(n, p)$	$\binom{n}{z}$	$\frac{p}{1-p}$	$(1+\Theta)^n - 1$	∞
$Poisson^*(\alpha)$	$\frac{1}{z!}$	α	$e^{\Theta} - 1$	∞
$Log(p)$	$\frac{1}{z}$	p	$-ln(1-\Theta)$	1
$Geom^*(p)$	1	$1-p$	$\frac{\Theta}{1-\Theta}$	1
$Pascal(k, p)$	$\binom{z-1}{k-1}$	$1-p$	$\left(\frac{\Theta}{1-\Theta}\right)^k$	1
$Bineg^*(k, p)$	$\binom{z+k-1}{z}$	p	$(1-\Theta)^{-k} - 1$	1

On the other hand, as Z is a r.v. discrete type for which $\mathbb{P}(Z \in \{1, 2, \ldots\}) = 1$, then we can write its distribution function:

$$F_Z(x) = \mathbb{P}(Z \leq x) = \begin{cases} 0 & \text{if} \quad x < 1 \\ \sum_{z=1}^{[x]} \mathbb{P}(Z = z) & \text{if} \quad x \geq 1. \end{cases}.$$

Therefore we can formulate the following proposition which characterizes the power series distribution class.

Proposition 2.1. *The necessary and sufficient condition for the r.v. Z to have a power series distribution is that the distribution function be characterized by:*

$$F_Z(x) = \sum_{z=1}^{[x]} \frac{a_z \Theta^z}{A(\Theta)}\ ,\ x \geq 1, \tag{2}$$

where the entities $(a_z)_{z\geq 1}$, Θ, $A(\Theta)$ are characterized by the Definition 2.1.

Proof. *Necessity*: assume that r.v. $Z \in PSD$. Then in accordance with Definition 2.1, this has the distribution

$$Z : \begin{pmatrix} 1 & 2 & 3\ldots & n\ldots \\ \frac{a_1\Theta}{A(\Theta)} & \frac{a_2\Theta^2}{A(\Theta)} & \frac{a_3\Theta^3}{A(\Theta)}\cdots & \frac{a_n\Theta^n}{A(\Theta)}\cdots \end{pmatrix}.$$

Then the distribution function $F_Z(x) = \mathbb{P}(Z \leq x) = \sum_{z\leq x} \mathbb{P}(Z = z) = \sum_{z=1}^{[x]} \frac{a_z\Theta^z}{A(\Theta)}$.

Sufficiency: assume that r.v. Z has the distribution function characterized by the relationship (2). With this we can restore the values of the r.v., namely $\mathbb{P}(Z \in \{1, 2, \ldots\} = 1)$, as well as the probabilities. We have that: $\mathbb{P}(Z = i) = F(i) - F(i-0) = \sum_{z=1}^{i} \frac{a_z\Theta^z}{A(\Theta)} - \sum_{z=1}^{i-1} \frac{a_z\Theta^z}{A(\Theta)} = \frac{a_z\Theta^z}{A(\Theta)}$, therefore $Z \in PSD$. ∎

The next result defines the distribution function for the maximum and minimum of a sequence of i.i.d.r.v. in random number.

Proposition 2.2. *If r.v. $U = \max\{X_1, X_2, \ldots, X_Z\}$ and $V = \min\{X_1, X_2, \ldots, X_Z\}$, where $(X_i)_{i\geq 1}$ are nonnegative i.i.d.r.v., with the known distribution function $F_{X_i}(x) = F(x)$, $\forall x > 0$ and $Z \in PSD$ with $\mathbb{P}(Z = z) = \frac{a_z\Theta^z}{A(\Theta)}$, $z = 1, 2, \ldots$; $\Theta \in (0, \tau)$, $\tau > 0$, r.v. $(X_i)_{i\geq 1}$ and Z being independent, then the distribution function of the r.v. U, respectively V are the following:*

$$U(x) = \frac{A\left[\Theta F(x)\right]}{A(\Theta)},\ x > 0, \tag{3}$$

$$V(x) = 1 - \frac{A\left[\Theta(1 - F(x))\right]}{A(\Theta)},\ x > 0. \tag{4}$$

Proof. For $Z = z$, the distribution function of the maximum of a sample of size z with the distribution function F, is $U_z(x) = [F(x)]^z$. Using the total probability formula, then the distribution function of the maximum of a sequence of i.i.d.r.v. in random number Z has the expression $U(x) = \sum_{z\geq 1} U_z(x) \cdot \mathbb{P}(Z = z) = \sum_{z\geq 1} [F(x)]^z \cdot \mathbb{P}(Z = z)$. Since $Z \in PSD$ (the relationship (1)), the result is relationship (3).

In the case of the minimum, we have $V(x) = \sum_{z\geq 1} V_z(x) \cdot \mathbb{P}(Z = z)$, where the distribution function of the minimum of a sample of size z, is characterized by the relationship $V_z(x) = 1 - [1 - F(x)]^z$.

Therefore,

$$\begin{aligned} V(x) &= \sum_{z\geq 1} [1 - [1 - F(x)]^z] \cdot \mathbb{P}(Z = z) \\ &= 1 - \sum_{z\geq 1} \frac{a_z [\Theta(1 - F(x))]^z}{A(\Theta)}. \end{aligned}$$

Taking into account the definition of $A(\Theta)$, relation (4) is obtained. ∎

The following results characterize the survival functions and the probability density functions (pdf) for the maximum, respectively minimum of a sequence of i.i.d.r.v. in random number.

Consequence 2.1. *If r.v.* $U = \max\{X_1, X_2, \ldots, X_Z\}$ *and* $V = \min\{X_1, X_2, \ldots, X_Z\}$, *where* $(X_i)_{i\geq 1}$ *are nonnegative i.i.d.r.v., with the known distribution function* $F_{X_i}(x) = F(x)$, $\forall x > 0$ *and* $Z \in PSD$ *with* $\mathbb{P}(Z = z) = \frac{a_z \Theta^z}{A(\Theta)}$, $z = 1, 2, \ldots;$ $\Theta \in (0, \tau)$, $\tau > 0$, *r.v.* $(X_i)_{i\geq 1}$ *and* Z *being independent, then the survival function of the r.v.* U, *respectively* V *are the following:*

$$S_U(x) = 1 - \frac{A[\Theta F(x)]}{A(\Theta)}, \ x > 0, \tag{5}$$

$$S_V(x) = \frac{A[\Theta(1 - F(x))]}{A(\Theta)}, \ x > 0. \tag{6}$$

Consequence 2.2. *If r.v.* $U = \max\{X_1, X_2, \ldots, X_Z\}$, *where* $(X_i)_{i\geq 1}$ *are nonnegative i.i.d.r.v., absolutely continuous type, with the known pdf* $f_{X_i}(x) = f(x)$, $\forall x > 0$ *and* $Z \in PSD$ *with* $\mathbb{P}(Z = z) = \frac{a_z \Theta^z}{A(\Theta)}$, $z = 1, 2, \ldots;$ $\Theta \in (0, \tau)$, $\tau > 0$, *r.v.* $(X_i)_{i\geq 1}$ *and* Z *being independent, then the pdf of the r.v.* U *is the following:*

$$u(x) = \frac{\Theta f(x) A'[\Theta F(x)]}{A(\Theta)}, \ x > 0. \tag{7}$$

Proof. The pdf of the r.v. U is obtained by determining the derivative of the relationship (3) with respect to variable x. ∎

Consequence 2.3. *If r.v.* $V = \min\{X_1,\ X_2, \ldots, X_Z\}$, *where* $(X_i)_{i\geq 1}$ *are nonnegative i.i.d.r.v., absolutely continuous type, with the known pdf* $f_{X_i}(x) = f(x),\ \forall x > 0$ *and* $Z \in PSD$ *with* $\mathbb{P}(Z = z) = \frac{a_z\Theta^z}{A(\Theta)},\ z = 1, 2, \ldots;\ \Theta \in (0, \tau),\ \tau > 0$, *r.v.* $(X_i)_{i\geq 1}$ *and* Z *being independent, then the pdf of the r.v. V is the following:*

$$v(x) = \frac{\Theta f(x) A'\ [\Theta(1 - F(x))]}{A(\Theta)},\ x > 0. \tag{8}$$

Proof. By determining the derivative of the relationship (4) with respect to variable x, (8) is obtained. ∎

In the above conditions, the hazard rate under definition, we can formulate the following proposition:

Proposition 2.3. *The hazard rate for the r.v. U, respectively V are characterized by the following relations:*

$$h_U(x) = \frac{u(x)}{1 - U(x)} = \frac{\Theta f(x) A'\ [\Theta F(x)]}{A(\Theta) - A\,[\Theta F(x)]},$$

and

$$h_V(x) = \frac{v(x)}{1 - V(x)} = \frac{\Theta f(x) A'\ [\Theta(1 - F(x))]}{A\,[\Theta(1 - F(x))]}.$$

The next result shows a characteristic of the distribution of the maximum of a random number of i.i.d.r.v. with the distribution function $F_{X_i}(x) \equiv F(x),\ x > 0$.

Proposition 2.4. *If* $(X_i)_{i\geq 1}$ *is a sequence of i.i.d.r.v., nonnegative, absolutely continuous type, with the distribution function* $F_{X_i}(x) \equiv F(x),\ x > 0$ *and* $Z \in PSD$ *with* $\mathbb{P}(Z = z) = \frac{a_z\Theta^z}{A(\Theta)}$, $(a_z)_{z\geq 1}$ *a sequence of nonnegative real numbers;* $A(\Theta) = \sum_{z\geq 1} a_z\Theta^z,\ \forall\ \Theta \in (0, \tau)$, *then r.v.* $U = \max\{X_1,\ X_2, \ldots, X_Z\}$ *has the limited distribution F if* $\Theta \to 0^+$. *In other words,*

$$\lim_{\Theta\to 0^+} U(x) = [F(x)]^k,\ x > 0$$

where $k = \min\{n \in \mathbb{N}^*,\ a_n > 0\}$.

Proof. By using the power series in (3) and by passing to the limit when $\Theta \to 0^+$, we obtain:

$$\lim_{\Theta\to 0^+} U(x) = \lim_{\Theta\to 0^+} \frac{A\,[\Theta F(x)]}{A(\Theta)} = \frac{0}{0}.$$

By applying the l ′ Hospital rule k-time, we have:

$$\begin{aligned}\lim_{\Theta\to 0^+} U(x) &= \lim_{\Theta\to 0^+} \frac{A^{(k)}\,[\Theta F(x)] \cdot [F(x)]^k}{A^{(k)}(\Theta)} \\ &= \frac{k!a_k\,[F(x)]^k}{k!a_k} = [F(x)]^k,\ x > 0\end{aligned}$$

and $k = \min\{n \in \mathbb{N}^*,\ a_n > 0\}$. ∎

Similarly, we can formulate a result regarding the distribution of the minimum of a sequence of i.i.d.r.v. in a random number.

Proposition 2.5. *Given the conditions the hypothesis of Proposition 2.4, r.v.* $V = \min\{X_1,\ X_2, \ldots, X_Z\}$ *has the limited distribution* F *if* $\Theta \to 0^+$, *or*

$$\lim_{\Theta \to 0^+} V(x) = 1 - [1 - F(x)]^l\,,\ x > 0$$

where $l = \min\{n \in \mathbb{N}^*,\ a_n > 0\}$.

The expressions of the r^{th} moments of the two distributions (maximum and minimum) can be derived as follows.

Proposition 2.6. *The* r^{th} *moment,* $r \in \mathbb{N},\ r \geq 1$ *of the r.v.* $U = \max\{X_1,\ X_2, \ldots, X_Z\}$, *where* $Z \in PSD$, *is given by:*

$$\mathbb{E}U^r = \sum_{z \geq 1} \frac{a_z \Theta^z}{A(\Theta)} \mathbb{E}\left[\max\{X_1,\ X_2, \ldots, X_z\}\right]^r. \tag{9}$$

Proof. By using the total probability formula with the help of the conditional mean, we obtain:

$$\begin{aligned} \mathbb{E}U^r &= \mathbb{E}\left(\mathbb{E}U^r\,|Z = z\right) = \sum_{z \geq 1} \mathbb{E}\left[\max\{X_1,\ X_2, \ldots, X_z\}\right]^r \cdot \mathbb{P}(Z = z) \\ &= \sum_{z \geq 1} \frac{a_z \Theta^z}{A(\Theta)} \cdot \mathbb{E}\left[\max\{X_1,\ X_2, \ldots, X_z\}\right]^r \end{aligned}$$

and this ends the proof. ∎

Similarly, we obtain the s^{th} moment, $s \in \mathbb{N},\ s \geq 1$ of the r.v. V:

$$\mathbb{E}V^s = \sum_{z \geq 1} \frac{a_z \Theta^z}{A(\Theta)} \mathbb{E}\left[\min\{X_1,\ X_2, \ldots, X_z\}\right]^s. \tag{10}$$

Consequence 2.4. *If* $(X_i)_{i \geq 1}$ *is a sequence of i.i.d.r.v., nonnegative, absolutely continuous type, with the distribution function* $F_{X_i}(x) \equiv F(x),\ x > 0$ *and pdf* $f_{X_i}(x) \equiv f(x),\ x > 0$, *then the* r^{th} *and* s^{th} *moments,* $r, s \in \mathbb{N},\ r, s \geq 1$ *of the r.v.* U, *respectively* V *are given by (9) and (10) where pdf ′ s* $f_{\max\{X_1, X_2, \ldots, X_z\}}(x) = zf(x)\left[F(x)\right]^{z-1}$ *and* $f_{\min\{X_1, X_2, \ldots, X_z\}}(x) = zf(x)\left[1 - F(x)\right]^{z-1}$.

3. SPECIAL CASES

This section presents examples of the distribution class of the maximum (U) and minimum (V) of a sequence $(X_i)_{i\geq 1}$ of i.i.d.r.v., nonnegative, of the absolutely continuous type. Briefly, let us call them the *Max-Poisson distribution* and the *Min-Poisson distribution*. The special cases are accompanied by expressions of the distribution function, the pdf, the hazard rate, the survival function, the mean and the variance.

3.1. THE MAX-POISSON DISTRIBUTION

In this subsection we present general distributions , which, when using concrete examples, can lead us to the complementary exponential Poisson distribution introduced by Cancho and others [3]. How $Z \sim Poisson^{\star}(\alpha) \in PSD,\ \alpha > 0$, and $A(\Theta) = e^{\Theta} - 1,\ \Theta = \alpha$, then the distribution function of the r.v. $U_{Poisson}$ is:

$$U_{Poisson}(x) = \frac{e^{\Theta F(x)} - 1}{e^{\Theta} - 1},\ x > 0, \tag{11}$$

and the pdf., hazard rate and survival function are given by:

$$u_{Poisson}(x) = \frac{\Theta f(x) e^{\Theta F(x)}}{e^{\Theta} - 1}, \tag{12}$$

$$h_{U_{Poisson}}(x) = \frac{\Theta f(x) e^{\Theta F(x)}}{e^{\Theta} - e^{\Theta F(x)}}$$

and

$$S_{U_{Poisson}}(x) = \frac{e^{\Theta} - e^{\Theta F(x)}}{e^{\Theta} - 1}.$$

The mean and variance of the Max-Poisson distribution are given by:

$$\mathbb{E}\ U_{Poisson} = \frac{1}{e^{\Theta} - 1} \sum_{z\geq 1} a_z \Theta^z \mathbb{E}\left[\max\{X_1,\ X_2, \ldots, X_z\}\right],$$

and

$$\begin{aligned} \mathbb{V}ar\ U_{Poisson} &= \frac{1}{e^{\Theta} - 1}\left[\sum_{z\geq 1} a_z \Theta^z \mathbb{E}\left[\max\{X_1,\ X_2, \ldots, X_z\}\right]^2 - \right. \\ &- \left. \frac{1}{e^{\Theta} - 1}\left(\sum_{z\geq 1} a_z \Theta^z \mathbb{E}\left[\max\{X_1,\ X_2, \ldots, X_z\}\right]\right)^2\right]. \end{aligned}$$

Particular case: if $X_i \sim Exp(\lambda),\ \lambda > 0$, then the relationships (11) and (12) lead to the distribution function introduced by Cancho and others [3]:

$$U_{Exp-Poisson}(x) = \frac{e^{-\alpha e^{-\lambda x}} - e^{-\alpha}}{1 - e^{-\alpha}}$$

and

$$u_{Exp-Poisson}(x) = \frac{\alpha\lambda e^{-\lambda x - \alpha e^{-\lambda x}}}{1 - e^{-\alpha}}.$$

3.2. THE MIN-POISSON DISTRIBUTION

A new general distribution is illustrated in this section given the fact that $V = \min\{X_1, X_2, \ldots, X_Z\}$, $Z \sim Poisson^{\star}(\alpha) \in PSD$, $\alpha > 0$, $A(\Theta) = e^{\Theta} - 1$, $\Theta = \alpha$, while $(X_i)_{i\geq 1}$ are i.i.d.r.v. absolutely continuous and nonnegative type.

Taking into account the relations (4) and (8) we obtain the distribution function, the pdf, the hazard rate and the survival function

$$V_{Poisson}(x) = \frac{e^{\Theta} - e^{\Theta(1-F(x))}}{e^{\Theta} - 1}, \tag{13}$$

$$v_{Poisson}(x) = \frac{\Theta f(x) e^{\Theta(1-F(x))}}{e^{\Theta} - 1}, \tag{14}$$

while

$$h_{V_{Poisson}}(x) = \frac{\Theta f(x) e^{\Theta(1-F(x))}}{e^{\Theta} - e^{\Theta(1-F(x))}}$$

and

$$S_{V_{Poisson}}(x) = \frac{e^{\Theta(1-F(x))} - 1}{e^{\Theta} - 1}.$$

The mean and variance of the Min-Poisson distribution are:

$$\mathbb{E}\, V_{Poisson} = \frac{1}{e^{\Theta} - 1} \sum_{z\geq 1} a_z \Theta^z \mathbb{E}\left[\min\{X_1, X_2, \ldots, X_z\}\right],$$

while

$$\begin{aligned} \mathbb{V}ar\, V_{Poisson} &= \frac{1}{e^{\Theta} - 1} \left[\sum_{z\geq 1} a_z \Theta^z \mathbb{E}\left[\min\{X_1, X_2, \ldots, X_z\}\right]^2 - \right. \\ &- \left. \frac{1}{e^{\Theta} - 1} \left(\sum_{z\geq 1} a_z \Theta^z \mathbb{E}\left[\min\{X_1, X_2, \ldots, X_z\}\right] \right)^2 \right]. \end{aligned}$$

Particular case: if $X_i \sim Exp(\lambda)$, $\lambda > 0$, then through the relationships (13) and (14), the distribution function and the pdf, obtained by Kus [6] are the following:

$$V_{Exp-Poisson}(x) = \frac{1 - e^{-\Theta F(x)}}{1 - e^{-\Theta}} = \frac{1 - e^{-\alpha\left(1 - e^{-\lambda x}\right)}}{1 - e^{-\alpha}},$$

and:

$$v_{Exp-Poisson}(x) = \frac{\Theta f(x) e^{-\Theta F(x)}}{1 - e^{-\Theta}} = \frac{\alpha\lambda e^{-\alpha-\lambda x+\alpha e^{-\lambda x}}}{1 - e^{-\alpha}}.$$

4. ON SOME DISTRIBUTIONS IN PROGRESS

We present in tabular form the different distributions that have been discussed and analyzed by some researchers in works [6] and [11], as well as an example of the new Gamma-Poisson distribution for the minimum and the Weibull-Poisson distribution for the maximum. The distribution function and the pdf are determined for each case.

Table 2: Distribution function and pdf of the r.v. $V_{Poisson}$ for different combinations

Authors	**Distribution**		**Distribution function**	**Probability density**
	Z	X_i		
Kus(2007)	$Poisson^{\star}(\alpha)$	$Exp(\lambda)$	$\frac{1-e^{\alpha\left(e^{-\lambda x}-1\right)}}{1-e^{-\alpha}}$	$\frac{\alpha\lambda e^{-\alpha-\lambda x+\alpha e^{-\lambda x}}}{1-e^{-\alpha}}$
Morais and Bareto-Souza (2011)		$Weibull(\lambda, \nu)$	$\frac{e^{\alpha}-e^{\alpha}e^{-\left(\frac{x}{\lambda}\right)^{\nu}}}{e^{\alpha}-1}$	$\frac{\alpha\frac{\nu}{\lambda}\left(\frac{x}{\lambda}\right)^{\nu-1}e^{-\left(\frac{x}{\lambda}\right)^{\nu}}e^{\alpha e^{-\left(\frac{x}{\lambda}\right)^{\nu}}}}{e^{\alpha}-1}$
New		$Gamma(a, b)$	$\frac{e^{\alpha e^{-bx}\sum_{i=0}^{a-1}\frac{(bx)^i}{i!}}}{e^{\alpha}-1}$	$\frac{\alpha b^a x^{a-1} e^{-bx+\alpha e^{-bx}\sum_{i=0}^{a-1}\frac{(bx)^i}{i!}}}{(a-1)!(e^{\alpha}-1)}$

Table 3: Distribution function and pdf of the r.v. $U_{Poisson}$ for different combinations

Authors	**Distribution**		**Distribution function**	**Probability density**
	Z	X_i		
Cancho and al (2011)	$Poisson^{\star}(\alpha)$	$Exp(\lambda)$	$\frac{e^{-\alpha e^{-\lambda x}}-e^{-\alpha}}{1-e^{-\alpha}}$	$\frac{\alpha\lambda e^{-\lambda x-\alpha e^{-\lambda x}}}{1-e^{-\alpha}}$
New		$Weibull(\lambda, \nu)$	$\frac{e^{-\alpha e^{-\left(\frac{x}{\lambda}\right)^{\nu}}}-e^{-\alpha}}{1-e^{-\alpha}}$	$\frac{\alpha\frac{\nu}{\lambda}\left(\frac{x}{\lambda}\right)^{\nu-1}e^{\alpha\frac{\nu}{\lambda}\left(\frac{x}{\lambda}\right)^{\nu-1}e^{-\left(\frac{x}{\lambda}\right)^{\nu}}-\left(\frac{x}{\lambda}\right)^{\nu}}}{e^{\alpha}-1}$

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