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Synthesis of the PID Algorithm for Models of Objects with Double Astatism and Dead Time

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***Abstract*-The paper summarizes the tuning algorithm for models of objects with inertia and astatism of the second degree and dead time, which describe the dynamics of various technical objects and technological processes. These models of tuned objects have the original double pole and a negative pole and an infinity of poly-zeros due to the dead time component. In order to tune the PID controller algorithm to the model of the given object, the algorithm was elaborated based on the analytical method of the maximum degree of stability. The dead time component approximates by the Pade approximants with nonminimal phase. For the approximate object model, the PID algorithm is synthesized using the maximum degree method with iterations. In order to verify the results obtained at the synthesis of the PID algorithm by the analytical method and method with iterations of the maximum degree of stability, the synthesis of the tuned algorithm was performed using the method of polynomial equations. An example of a system with the control object model and the controller synthesized according to these methods with computer simulation in the MATLAB package was examined and the system performance was analyzed. The advantages of the method of the maximum degree of stability with iterations through reduced calculations and minimum time are highlighted, which lead to the simplification of the procedure for tuning the PID algorithm for these object models and higher system robustness.**

Keywords- Model of the tuned object with inertia; double astatism and deadtime; transfer function, PID algorithm; tuning of the controller parameters; method of maximum degree of stability with iterations; system performance

I. INTRODUCTION

In the practice of automation there are a variety of technical objects such as automobile, spacecraft, rocket, telescope, plotter, laser, elevator, nuclear reactor electrode, linear drives, etc., industrial and technological processes, which require automatic control [1, 2] and which are described by the mathematical models with inertia, double integration and dead time presented with the transfer functions of the form:

$$H(s)=\frac{ke^{-ds}}{s^2(Ts+1)}=\frac{ke^{-ds}}{Ts^3+s^2}=\frac{ke^{-ds}}{a_0s^3+a_1s^2}, \qquad (1)$$

where k is the transfer coefficient, T - the inertia time constant, d - the dead time, and $a_0=T$, $a_1=1$ are the generalized coefficients.

The presence in the model of the control object (1) of the second degree astatism and dead time raises difficult problems, when tuning the PID controller to these models. Several methods for tuning controllers such as the Ziegler-Nichols method, the poly-zero method, the frequency method, etc. can not be applied or are difficult [1-4].

In the paper it was proposed to use the method of analytical maximum degree of stability (AMSD) and the GMS method with iterations (MSDI) of the automatic system for the synthesis of the PID algorithm [7-10].

II. PID CONTROLLER TUNING ALGORITHMS

The study uses the structural scheme-block of the automatic system (AS) consisting of the controller with transfer function $H_R(s)$ and the model of the object with transfer function $H_P(s)$ as given in Fig. 1.

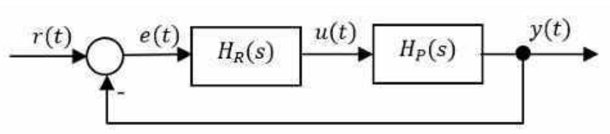

Figure 1. Structural block scheme of the automatic system.

The standard PID tune algorithm is described with the transfer function:

$$H_{PID}(s)=k_p+\frac{k_i}{s}+k_ds=\frac{k_ds^2+k_ps+k_i}{s}, \qquad (2)$$

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where k_p, k_i, k_d are the tuning parameters of the PID algorithm.

Applying the method of the maximum degree of stability of the system, the calculation expressions of the PID tuning parameters for the model of the control object (1) are presented in the form [8-10]:

$$-d^3a_0J^4+(12d^2a_0+d^3a_1)J^3-(36da_0+9d^2a_1)J^2+ \\ +(24a_0+18da_1)J-6a_1, \quad (3)$$

$$k_d=\frac{e^{-ds}}{2k}(-d^2a_0J^4+(8da_0+d^2a_1)J^3-(12a_0+ \\ +6da_1)J^2+6a_1J)=f_d(J), \quad (4)$$

$$k_p=\frac{e^{-ds}}{k}(-da_0J^4+(4a_0+da_1)J^3-3a_1J^2)+ \\ +2k_dJ)=f_p(J), \quad (5)$$

$$k_i=\frac{e^{-ds}}{k}(-a_0J^4+a_1J^3)-k_dJ^2+k_pJ)=f_i(J), \quad (6)$$

At the known values of the model of the tuned object (1) the optimal degree of stability J from (3) is determined and using the expressions (4)-(6) the optimal parameters $k_p=f_p(J)$, $k_i=f_i(J)$, $k_d=f_d(J)$ of the PID algorithm are calculated.

The maximum degree of stability J is the smallest value of the real root or the real part of the complex root of the equation (3).

If the values of k_p, k_i, k_d for the optimal parameters of the PID algorithm do not satisfy the performance of the system, it is proposed to use the MSD method with iterations for calculus.

Variation $J=0...\infty$ are calculated and then are constructed the curves (4)-(6) $k_p=f_p(J)$, $k_i=f_i(J)$, $k_d=f_d(J)$ for the PID algorithm. On these curves $k_p=f_p(J)$, $k_i=f_i(J)$, $k_d=f_d(J)$ is choosen a set of values of the parameters of the PID controller J_i - k_{pi}, k_{ii}, k_{di} and it is simulated on the computer the automatic system and are rised the transient response of the system after which the highest performances of the system are determined.

III. Simulation and Analysis of an Example

The parameter values of the tuned object model are considered to be known for two cases:

1) transfer coefficient $k=1$, time constant $T=0.1s$, dead time $d=0.5s$;
2) transfer coefficient $k=0.5$, time constant $T=10s$, dead time $d=2s$.

The performance of the automatic system is required: the stationary error $e=\pm5\%$ of the stationary value of the output value y_{st}, the rise time $t_c=2s$, the settling time $t_r=10s$ and the overshoot $w=10\%$.

For the object model (1) it is proposed to synthesize the PID controller algorithm tuned according to the analytical MSD method and MSD method with iterations.

It is proposed to analyze the performance and robustness of the automated system with the PID controller given by the analytical MSD method and the MSD method with iterations to the variation of the object model parameters by ±50% from the model values to step signal perturbation action.

The analytical MSD method is used for the case when the parameters of the object model have the values $k=1$, $T=0.1s$, $d=0.5s$, it is determined the maximum degree J from (3) and is calculated the optimal values of the parameters k_p, k_i, k_d according to the expressions (4)-(6), which are given in table 1, row 1. The automatic system was simulated on the computer in MATLAB and the transient response is given in Fig. 2, *a*, curve 1, and the performances are given in Table I, row 1.

For the model (1) with parameters $k=1$, $T=0.1s$, $d=0.5s$, the PID controller was tuned using the parametric optimization method in MATLAB and the controller parameters are given in Table I, row 2, the transient process is given in Fig. 2, *a*, curve 2, and the performances are presented in Table I, row 2.

TABLE I. Controller Parameters and Performance of Automatic System

Nr.	Model parameters	Controller parameters				Performances of the system			
		J	k_p	k_i	k_d	t_c	w	t_r	n
1	Nom. 1A	0.6784	0.3374	0.0536	0.8315	1.62	58.27	11.95	2
2	PO	Not applicable							
3	T^+	0.6784	0.3374	0.0536	0.8315	1.66	63.79	11.57	2
4	T^-	0.6784	0.3374	0.0536	0.8315	1.58	53.09	12.25	2
5	k^+	0.6784	0.3374	0.0536	0.8315	1.29	73.17	3.39	1
6	k^-	0.6784	0.3374	0.0536	0.8315	Oscillating process			
7	d^+	0.6784	0.3374	0.0536	0.8315	1.82	92.49	9.31	2

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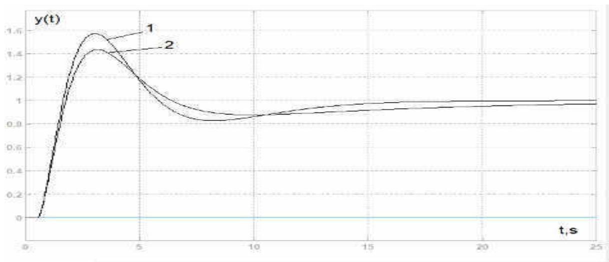
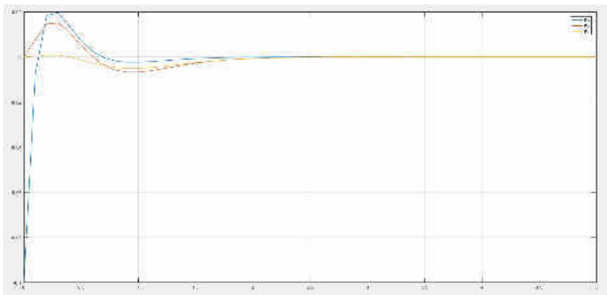
8	d^-	0.6784	0.3374	0.0536	0.8315	1.54	39.19	13.11	2
9	Nom. 1I	0.9000	0.2571	0.0203	0.7828	1.74	44.4	7.37	1
10	T^+	0.9000	0.2571	0.0203	0.7828	1.76	48.76	7.09	1
11	T^-	0.9000	0.2571	0.0203	0.7828	1.71	40.47	7.6	1
12	k^+	0.9000	0.2571	0.0203	0.7828	1.35	58.24	6.92	2
13	k^-	0.9000	0.2571	0.0203	0.7828	Oscillating process			
14	d^+	0.9000	0.2571	0.0203	0.7828	1.91	71.31	8.26	1
15	d^-	0.9000	0.2571	0.0203	0.7828	1.71	30.61	8.26	2

The MSD method with iterations is applied for the case when the parameters of the object model have the values $k=1$, $T=0.1s$, $d=0.5\,s$ and the independent variable J was varied. Using relations (4)-(6) the calculas of curves $k_p=f_p(J)$, $k_i=f_i(J)$, $k_d=f_d(J)$ parameters of the controller were performed and they are given in Fig. 2, *b*.



a)



b)

Figure 2. Transient responses a) and curves: $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ b) of the automatic system.

On these curves, sets of values J_i - k_{pi}, k_{ii}, k_{di} were chosen (three data variants were analyzed and in table 1, row 8 is given the variant for the highest performances) and using this values the automatic system was simulated on the computer in MATLAB with the parameters of the PID controller according to Table I, row 9 and the transient response is presented in Fig. 2, *a*, curve 3, and the performances of the automatic system are given in Table I, row 9.

The analytical MSD method is also used for the case when the parameters of the object model have the values $k=0.5$, $T=10s$, $d=2s$, it is determined the maximum degree J from (3) and it is calculate the optimal values of the parameters k_p, k_i, k_d also using the expressions (4)-(6), which are given in Table II, row 1. The automatic system was simulated on the computer in MATLAB and the results for transient response is given in Fig. 3, *a*, curve 1, and the performances are given in Table II, row 1.

For model (1) with parameters $k=0.5$, $T=10s$, $d=2s$ the PID controller was tuned using the parametric optimization method in MATLAB and the controller parameters are given in Table II, row 2, the transient process is given in Fig. 3, *a*, curve 2, and the performances are presented in Table II, row 2.

TABLE II. CONTROLLER PARAMETERS AND PERFORMANCE OF AUTOMATIC SYSTEM

Nr.	**Model parameters**	**Controller parameters**				**Performances of the system**			
		J	k_p	k_i	k_d	t_c	w	t_r	n
1	Nom. 2A	0.0232	0.00101	5.61	0.0671	41.79	42.70	310.09	2
2	PO	Not applicable							
3	T^+	0.0232	0.00101	5.61	0.0671	44.69	54.14	289.39	2
4	T^-	0.0232	0.00101	5.61	0.0671	40.19	33.13	325.00	2
5	k^+	0.0232	0.00101	5.61	0.0671	26.69	43.32	95.79	1
6	k^-	0.0232	0.00101	5.61	0.0671	Oscillating process			
7	d^+	0.0232	0.00101	5.61	0.0671	41.99	45.10	306.49	2

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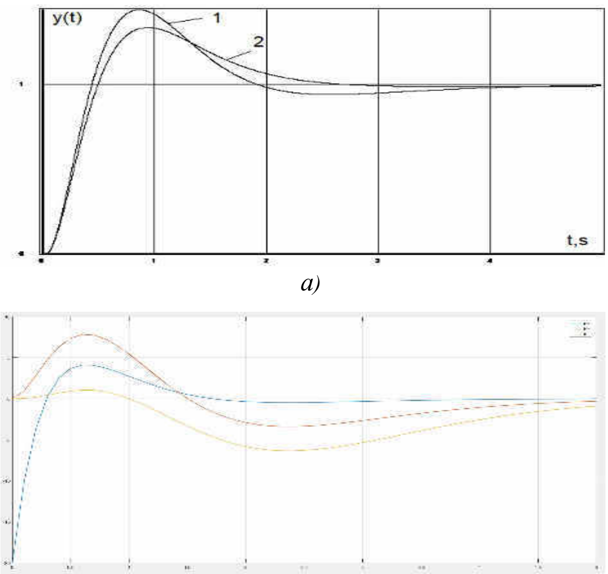
8	d^-	0.0232	0.00101	5.61	0.0671	41.79	39.52	317.19	2
9	Nom. 2A	0.03	0.00073	1.793	0.0618	46.67	35.36	211.19	1
10	T^+	0.03	0.00073	1.793	0.0618	49.31	41.65	191.51	1
11	T^-	0.03	0.00073	1.793	0.0618	45.83	25.6	226.91	1
12	k^+	0.03	0.00073	1.793	0.0618	35.03	32.16	166	1
13	k^-	0.03	0.00073	1.793	0.0618	Oscillating process			
14	d^+	0.03	0.00073	1.793	0.0618	46.67	34.30	207.29	1
15	d^-	0.03	0.00073	1.793	0.0618	46.79	30.97	214.31	1

The MSD method with iterations is applied when the parameters of the object model have the values $k = 0.5$, $T = 10s$, $d = 2s$ and the independent variable J was varied and using the relations (4)-(6) the calculations of the curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ parameters of the controller were performed and the result are given in Fig. 3, *b*. On these curves, sets of values J_i - k_{pi}, k_{ii}, k_{di} were chosen (three data variants were analyzed and in Table II, row 9 is given the variant for the highest performances), the automatical system was simulated on the computer in MATLAB with the set of parameters for the PID controller according to Table II, row 9 and the transient process is presented in Fig. 3, *b*, curve 3, and the performances of the automatic system are given in Table II, row 9.



a)

b)

Figure 3. Transient responses a) and curves: $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ b) of the automatic system.

The polynomial method is used to compare the results obtained by applying the analytical MSD method and MSD method with iterations when tuning the PID controller to the object model (1) [5,6] and calculations have shown that this method does not apply.

For examples 1) and 2) the values of the model parameters (1) were varied and the automated system with PID controller tuned according to analytical MSD method and MSD method with iterations was simulated in MATLAB and the systems performances are given in tables 1 and 2, rows 3-7 and 9-15 respectively.

The parametric optimization method for both examples is not applicable.

For the automatic system with the object model with the parameters $k = 1$, $T = 0.1s$, $d = 0.5s$ and with the PID controller tuned according to the analytical MSD method with increasing T by 50% $T^+ = 0.15$ the time value t_c increases 2.5 times and the value of settling time t_r decreases 1.03 times and overshoot value increases 1.09 times. In case when the value T is reduced by 50% $T^- = 0.05$, the time value t_c decreases 2.5 times and the time value t_r increases 1.025 times, and the overshoot value is reduced 1.09 times.

With the increase of value k by 50% $k^+ = 0.75$ the time values t_c and t_r are reduced by 1.26 times and 3.53 times respectively and the overshoot value increases by 1.26 times. In case of reduction of k by 50% $k^- = 0.25$ the process becomes very oscillating. With increasing dead time d by 50% $d^+ = 3$ the time value t_c increases 1.12 times, t_r decreases 1.26 times and the overshoot value increases 1.59 times. When the value of d is reduced by 50%, the value $d^- = 0.05$, time value t_c decreases 1.05 times, t_r increases 1.1 times and the overshoot value is reduced 1.49 times.

For the automatic system with the object model with the parameters $k = 1$, $T = 0.1s$, $d = 0.5s$ and with the PID controller tuned according to the GMS method with iterations with increase of T by 50% $T^+ = 0.15$, the

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time value t_c remains unchanged, the settling value t_r is reduced by 1.04 times and the overshoot value is reduced by 1.12 times. In case when value T is reduced by 50% $T^- = 0.05$, the time value t_c remains unchanged, the settling value t_r increases 1.03 times, and the overshoot value is reduced 1.14 times.

With the increase of k by 50% $k^+ = 0.75$, the time values t_c and t_r increase 1.29 times and 1.07 times respectively and the overshoot value increases 1.31 times. With the reduction of k by 50% $k^- = 0.25$ and the process becomes very oscillating. With the increase of the dead time by 50% $d^+ = 3$ the time values t_c and t_r are reduced 1.1 times and 1.12 times respectively and the overshoot value increases 1.61 times. In case of reduction d by 50% $d^- = 0.05$, the time values t_c and t_r practically remain unchanged and the overshoot value is reduced by 1.45.

For the automatic system with the object model with the parameters $k = 1$, $T = 0.1s$, $d = 0.5s$ and with the PID controller tuned according to the MSD method with iterations the performance are higher: the settling time t_r by 1.62 times and the overshoot by 1.31 times than the system performance with the controller tuned according to the analytical MSD method.

For the case of the automatic system with the object model with parameters $k = 0.5$, $T = 10s$, $d = 2s$ and with the PID controller tuned according to the MSD method with iterations also the performances are higher: settling value t_r by 1.47 times and overshoot value by 1.21 times than the system performances with the controller tuned according to the analytical MSD method.

At the action of the perturbation $p(t) = \pm 1(t)$ on the object with $T = 0.1s$ and $T = 10s$ the transient response of the system is restored during the settling time.

IV. Conclusions

Analyzing the results obtained when tuning the PID algorithm to model (1) according to the MSD method with iterations, it is found:

- For the automatic system with the object model with the parameters $k = 1$, $T = 0.1s$, $d = 0.5s$ and with the PID controller tuned according to the MSD method with iterations the performance are higher: the settling time t_r by 1.62 times and the overshoot by 1.31 times than the system performance with the controller tuned according to the analytical MSD method.
- For the case of the automatic system with the object model with parameters $k = 0.5$, $T = 10s$, $d = 2s$ and with the PID controller tuned according to the MSD method with iterations also the performances are higher: settling value t_r by 1.47 times and overshoot value by 1.21 times than the system performances with the controller tuned according to the analytical MSD method.

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