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Tuning the PID Controller to the Object Model with Second-Order Inertia with Identical Elements and Time Delay by the Modified Polynomial Method

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***Abstract*—The paper presents the procedure for tuning the PID control algorithm to the object model with second-order inertia with identical elements and time delay according to the modified polynomial method. Methods that can be applied for tuning the PID control algorithm to this control object model are analyzed. The modified polynomial method of tuning the PID algorithm to the second-order inertial control object model with identical elements and time delay is developed, which presents as a simple procedure. To compare the obtained results, tuning methods are applied: the maximum stability degree method in analitycal form and with iterations, Ziegler-Nichols method and parametrical optimization of the PID controller to the model of the given object. The tuning algorithm according to the method of the maximum degree of stability with iterations and the modified polynomial method is synthesized for an example of the object model with second-order inertia with identical elements and time delay, and the results obtained for the variation of the object model parameters are analyzed. The advantages of the maximum stability degree methods with iterations and modified polynomial are highlighted.**

Keywords – second-order inertial object model with identical elements and time delay; tuning methods; PID algorithm; maximum stability degree method with iterations; modified polynomial method; performance; robustness

I. Introduction

In the automation of slow and very slow industrial and technological processes after the step response of the system the mathematical model of the control object is approximated. The paper analyzes the approximation of the step response of these processes with the transfer function with second-order inertia with identical elements and time delay, described by the transfer function:

$$H_P(s) = \frac{ke^{-ds}}{(Ts+1)^2}, \quad (1)$$

where k is the transfer coefficient, T – the time constant, d - the time delay.

The time delay component is a transcendent function, which has a strong influence on the stability and quality of the synthesized automatic system [1-2].

Dead-time transfer elements do not have finite-dimensional systematic realizations, but have an infinite number of poly-zeros, and to obtain rational representations the transcendental component is approximated by Pade approximants with minimum and non-minimum phase etc. [1-2].

Next it is necessary to tune the PID controller to the control object model (1). Methods for synthesizing control algorithms are developed: frequency, experimental methods, integral criteria etc. [1-7].

The frequency methods for synthesis the controllers are applicable for the model (1), but the calculations are accompanied by graphical constructions and the procedure becomes difficult [1-2].

The basic experimental method presents the Ziegler-Nichols (ZN) method, which is widely used in the practice of tuning the typical P, PI, PID algorithms to the model of object (1), but the system performances are reduced [1].

For the use of the integral criteria method, it is necessary that the time delay component be approximated with rational functions, which leads to raising the order of the object model and create difficulties in synthesizing the control algorithm.

The paper uses maximum stability degree method in analytical form (AMSD) [8], maximum stability degree method with iterations (MSDI) [9-10] and the polynomial method [2] for the tuning the PID controller to the control object (1).

An example of tuning the PID controller to the second-order inertia control object model with identical elements and time delay is presented, and the variation of the control object parameters from the nominal values is analyzed.

II. PID CONTROLLER TUNING ALGORITHMS

In the study, the structural block diagram of the automatic control system is used, consisting of the object model with transfer function $H_P(s)$ and the controller with transfer function $H_R(s)$ as it is shown in Figure 1, subjected to the action of the unit step input $r(t)$ and $e(t)$ is the system error, $u(t)$ – the command developed by the controller and $y(t)=h(t)$ – the step response of the system.

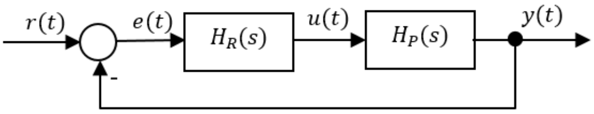

Figure 1. Structural block scheme of the automatic system.

Consider the standard PID control algorithm in parallel connection described with the transfer function:

$$H_{PID} = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \quad (2)$$

where k_p, k_i, k_d are tuning parameters of the proportional, derivative and integrative component of the PID control algorithm.

To tune the parameters of the PID control algorithm to the model of object (1), the method AMSD, MSDI methods and the polynomial method and the modified polynomial method are used.

For the method of the maximum stability degree in analitical form, the analytical expressions for calculations the parameters of the PID controller to the model of the control object (1) for the order of the model n are presented in the general form [9-10]:

$$(1-TJ)^n(-c_0J^4 + c_1J^3 - c_2J^2 + c_3J - c_4) = 0 \quad (3)$$

$$c_0 = d^3T^3,\ c_1 = 3d^2T^2(T(n+1)+d),$$
$$c_2 = 3dT(d^2 + dT(2n+3) + T^2n(n+1)),$$
$$c_3 = d^3 + 3d^2T(n+3) + 3dT^2n(n+3) + T^3n(n^2-1),$$
$$c_4 = 3d^2 + 6ndT + 3nT^2(n-1).$$

$$k_p = \frac{e^{-dJ}}{k}(1-TJ)^{n-2}(d^2T^2J^4 + dT(T(2n+1)+ +2d)J^3 + (d^2 + 2dT(n+1) + T^2(n^2-1))J^2 - -(d+T(n-2))J - 1) = \frac{e^{-dJ}}{k}(1-TJ)^{n-1}(-dTJ^2 + +(T(n+1)+d)-1)J + 2k_dJ = f_p(J) \quad (4)$$

$$k_i = \frac{e^{-dJ}}{2k}(1-TJ)^{n-2}(d^2T^2J^2 - 2dT(d+nT)J + +d^2 + 2dTn + T^2n(n-1))J^3 = =\frac{e^{-dJ}}{k}(1-TJ)^nJ - k_dJ^2 + k_pJ = f_i(J) \quad (5)$$

$$k_d = \frac{e^{-dJ}}{2k}(1-TJ)^{n-2}(d^2T^2J^3 - 2dT(d+ +T(n+1))J^2 + (d^2 + 2dT(n+2) + +T^2n(n+1))J - 2(d+Tn)) = f_d(J), \quad (6)$$

where J is the degree of stability of the system.

From the algebraic equation (3), the maximum degree of stability J_{opt} of the system is determined as the smallest after the positive real root value or the positive real part of the complex root [8-10].

Knowing the parameters of the object model (1) and the optimal degree of stability J_{opt}, from (4)-(6) the numerical values of the optimal parameters k_p, k_i, k_d of the PID controller are calculated.

Determining the parameters of the PID algorithm according to relations (4)-(6) does not guarantee the automatic system the stability and the highest performance of the designed system.

In this case, the maximum stability degree method with iterations is applied, which reduces to the following procedure. According to relations (4)-(6) as functions $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ of the degree of stability argument J, the argument $J = 0...\infty$ is varied and the curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ are constructed. Next, on these curves, sets of J_i values for $k_{pi} = f_p(J_i)$, $k_{ii} = f_i(J_i)$, $k_{dl} = f_d(J_i)$ are chosen iteratively on the slope of the curves in the vicinity of the optimal value of J_{opt}, the automatic system is simulated and the possible high performances of the automatic system are determined [9-10].

The polynomial method of tuning the PID controller to the object model (1) is based on the following arguments [2,6]. From the formulation of the controll algorithm synthesis procedure, the formula is applied to determine the transfer function of the controller $H_R(s)$ [1,2]:

$$H_R(s) = \frac{H_0(s)}{1-H_0(s)}\frac{1}{H_P(s)} = \frac{H_0(s)}{1-H_0(s)}\frac{(Ts+1)^2}{e^{-ds}}\frac{1}{k}, \quad (7)$$

where $H_0(s)$ is the transfer function of the unknown closed system, $H_P(s)$ is the transfer function of the controller.

1. It is considered that tuned automatic system ideally reproduces the unit step input signal and $H_0(s) = 1$.

2. The transcendental function e^{-ds} is approximated, applying the series presentation and keeping only the first two terms:

$$e^{-ds} \approx 1 - ds. \quad (8)$$

Based on conditions (8), relation (7) is presented in the following form:

$$H_R(s) = \frac{H_0(s)}{1-H_0(s)}\frac{1}{H_P(s)} = \frac{1}{1-1}\frac{(Ts+1)^2}{e^{-ds}}\frac{1}{k} =$$
$$= \frac{1}{1-(1-ds)}\frac{T^2s^2+2Ts+1}{k} = \frac{T^2s^2+2Ts+1}{kds} =$$
$$= \frac{T^2s^2}{kds} + \frac{2Ts}{kds} + \frac{1}{kds} = k_p + \frac{1}{T_i s} + k_d. \quad (9)$$

where $k_p = 2T/kd$ is the coefficient of the proportional part, $T_i = kd$ represents the integration time constant and $k_i = 1/T_i$, $k_d = T_d = T^2/kd$ are the derivation time constant.

Relation (9) shows the proportional-integrative-derivative PID controller with parameters k_p, k_i and k_d.

After an analysis of the stability conditions and the simulation of the automatic system for the case of the model (1), the solution is adopted to introduce the weighting coefficient n in the formula (9) for calculating the controller parameters according to the modified polynomial method to satisfy the stability and performance conditions of system:

$$k_p = \frac{T}{nkd}, \; T_i = nkd, \; k_i = \frac{1}{nkd}, \quad (10)$$

where $n = 2...4$ depending on the desired performance of the automatic system.

III. Applications and Simulation on the Computer

Let's consider the model of the controller object with inertia of the first order with identical elements and time delay with the parameters: transfer coefficient $k = 2$, time constant $T = 10\,s$, time delay $d = 2\,s$.

Calculations are performed to tune the PID algorithm to the model (1) according to the AMSD method. From (3) as an algebraic equation of the fourth degree, the optimal stability degree $J_{opt} = 0.2728$ was obtained and the parameters of the PID controller were calculated according to relations (4)-(6), which are presented in row one of Table I. After an analysis with the GMSI method, the degree of stability $J_{opt} = 0.2728/2 = 0.1364$ was determined and the parameters of the PID controller presented in row two of Table I were determined, and the calculation of the parameters of the PID controller for $J = 0.14$ is given in row three of Table I.

TABLE I. THE TUNING PARAMETERS AND AUTOMATIC CONTROL SYSTEM PERFORMANCE

No	**Method**	**Max. degree *J***	**Tuning parameters**				**Performance of the system**			
			k_p	k_i, s^{-1}	T_i, s	k_d, s	t_c, s	c, %	t_r, s	n
1	AMSD	0.2728	2.9834	0.217	2.9394	11.195	6.49	49.17	30.81	3
2	MSDI1	0.1364	1.5275	0.0828	12.0773	11.0857	10.83	-	10.83	-
3	MSDI2	0.14	1.595	0.0874	11.4416	1.17	10.17	3.14	10.17	-
4	PM		5	0.25	4.000	25	Unstable system			
5	PM1 $n=2$		2.5	0.125	8.00	12.5	6.59	29.05	21.15	2
6	PM2 $n=3$		1.6667	0.0833	12.0048	8.3333	9.37	2.75	9.37	-
7	ZN		3.186	0.0822	12.165	2.535	8.27	46.06	55.77	4
8	OP		1.35	0.064	15.62	6.5447	15.82	-	15.82	-

The calculations according to (10) were performed to assign the parameters of the PID controller for the polynomial method, the modified polynomial method for two values of the weighting coefficient $n=2$ and 3 and the results are shown in Table I: row 4 polynomial method MP, row 5 method modified polynomial PM1 and row 6 modified polynomial method PM2.

To compare the results obtained by the MSDI and modified polynomial methods, the known methods of tuning the PID algorithm to the given model are applied and the results are presented in Table I: row 6 ZN method, row 7 – parametric optimization method.

According to the data in Table I, the automatic system with the given PID controller was simulated on the computer in MATLAB and some of the responses are shown in Figure 2 for the methods: AMSD - curve 1, MSDI1 with $J=0.1364$ – curve 2, modified polynomial PM2 – curve 3 and the parametric optimization method – curve 4 and the performances for all simulated systems are also presented in Table I.

With increasing k, the system with the controller tuned according to the MSDI method has 1.41 times less overshoot and 1.42 times less settling time, than the system with the controller tuned according to the modified polynomial method, and with decreasing k the systems have the same robustness (aperiodic processes).

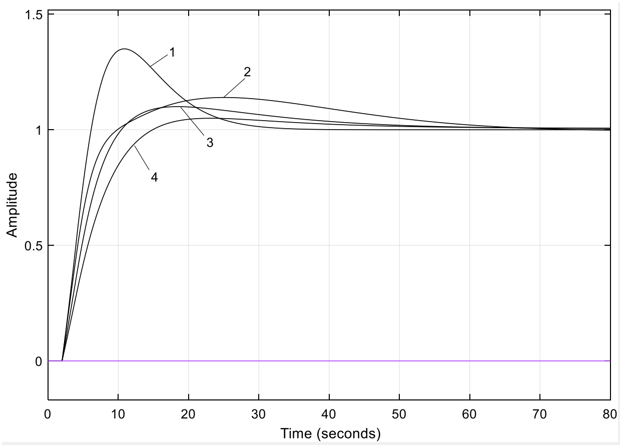

Figure 2. Response of the automatic system

Next, the parameters k, T, d of the object model (1) were changed by $\pm 50\%$ from the nominal values and the system was simulated with the tuned controller according to the MSDI method and the modified polynomial method (PM) at unit step input and the data are presented in Table II.

TABLE II. THE PERFORMANCE OF THE AUTOMATIC CONTROL SYSTEM AT THE VARIATION PARAMETERS OF THE MODEL OF CONTROL OBJECT

Method	**Variation parameters of the object**			**Performance of the system**			
	$k=2$	$T=10$	$d=2$	t_c, s	c, %	t_r, s	n
MSDI1	3/1	10/10	2/2	7.06/30.14	18.09/0	14.22/30.14	1/0
	2/2	15/5	2/2	16.68/4.55	17.70/45.0	50.5/60	1/11
	2/2	10/10	3/1	10.30/14.15	12.65/0	19.55/14.15	1/0
PM2	3/1	10/10	2/2	6.51/32.93	25.60/0	20.29/32.93	2/0
	2/2	15/5	2/2	15.60/oscil	14.30/oscil	43.87/oscil	1/oscil
	2/2	10/10	3/1	9.38/14.06	17.81/0	26.30/14.06	2/0

With the increase of T, the system with the controller tuned according to the modified polynomial method PM2 has higher performances: the rise time t_c by 1.06 times, the overshoot n by 1.24 times and the settling time t_r by 1.15 times, and with the reduction of T, the system with the controller tuned by the MSDI1 method is much more robust than the PM2 modified polynomial tuned controller system, which destabilizes – highly oscillating responses.

With the increase of d , the system with the controller tuned according to the MSDI1 method is 1.41 times more robust after overshoot and 1.34 times after the settling time, than the system with the controller tuned according to the PM method, and with the reduction of *k* the systems have the same robustness (aperiodic processes).

IV. CONCLUSIONS

1. For tuning the PID controller to the second-order inertial object model with identical elements with time delay with the parameter values from the analyzed example, it is recommended to use the modified polynomial method, for which the calculations are very simple.

2. The system with the controller tuned according to the MSDI1 method and the system with the controller tuned according to the modified polynomial method PM2 within the limits of the 5% steady state error have the same performance. The system with the tuned controller according to the modified polynomial method PM2 has destabilizing tendencies.

3. When the parameters of the object model in the analyzed example (Table II) are varyed, the automatic system with the controller tuned according to the MSDI1 method is more robust than the automatic system with the controller tuned according to the modified polynomial method PM2.

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