KALMAN FILTER TO DETERMINE ORIENTATION AND ATTITUDE OF MICROSATELLITE

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Abstract: *The nonlinear problem of tracking and predicting where a satellite will be over some time can be difficult with the recognition of modeling error and ground site radar tracking errors. For this reason it is important to have an accurate modeling program with the fidelity to correct for any errors in orbital motion and predict the most accurate positioning at some future time. The Extended Kalman Filter is one such program that can accurately determine position over time given estimate ranges for sources of error. However, the Extended Kalman Filter contains many linear approximations that allow its prediction and correction methods to work. This paper will discuss the effects of replacing the linearizing approaches made in the orbital model part of the program with numerical small-step approaches.*

Keywords: *Kalman Filter, approximation, prediction, estimation, correction, satellite.*

1. Introduction

The application of satellites takes many forms including international communications and television relay to space-based telescopes. However, a satellite's effectiveness is directly based on the ability to track and communicate with the spacecraft over its lifetime. In order to track a satellite, a ground station will use a site-defined coordinate frame to track the motion. This coordinate frame uses the distance from the site to the satellite, the azimuth angle, and the elevation angle to determine where in the sky a satellite is at some time. The ability for a ground station site to find and track a satellite is based of the predictions made by mathematical models of the orbit. The more accurate a model can be at predicting a satellite's motion, the more successful a ground station will be at finding and tracking the satellite.

2. Kalman Filter

The Kalman Filter is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms. It estimates the state of a dynamic system. This dynamic system can be disturbed by some noise, mostly assumed as white noise. To improve the estimated state the Kalman filter uses measurements that are related to the state but disturbed as well.

Thus the Kalman filter consists of two steps:

1. The prediction
2. The correction

In the first step the state is predicted with the „dynamic model". In the second step it is corrected with the „observation model", so that error covariance of the estimator is minimized. This procedure is reapeted for each time step with the state of the previous time step as initial value.

3. The Extended Kalman Filter

Small perturbations felt by an orbiting body cannot be fully modeled. Additionally errors in the method of obtaining the actual position and velocity from radar data will cause errors. For these reasons it is essential to use a filtering device such as the Kalman Filter to statistically determine the most probable position of a satellite. This method also contains some errors in its prediction algorithms that take the form of mathematical linearizations. By seeking methods of reducing these linearizations this paper explores methods of improving the Extended Kalman Filter for better overall prediction and orbit determination. This approach has not been used before in the Extended Kalman Filter.

Because there are only a limited number of ground stations around the Earth, a satellite cannot be continuously tracked. Once a connection is lost, only the prediction of where the satellite will be at some later time can help in reestablishing communication. One of the most accurate methods of establishing a model that takes the orbital determination errors into account is the Extended Kalman Filter.

The Extended Kalman Filter (EKF) is a stochastic estimation algorithm. The EKF simply uses a weighted statistical average of the difference in position and velocity inputs predicted from the model and known from the ground site to correct the model towards more precise predictions based on the known errors in those inputs.

The EKF can be tuned to use anything from the basic two-body orbit model to highly accurate multi-perturbation model for its predictions. The effects of recognizing initial modeling error can also be explored making the EKF a valuable orbit prediction and modeling tool. Figure 1 shows the big picture of how the EKF is used in orbital modeling. Observations from one site allow for continual orbital model refinement and prediction to another site.

The normal Kalman Filter was created for linear applications. Because many systems like orbital motion are nonlinear, the extended filter makes many approximations to reduce nonlinear systems to linear models. For these reasons, the EKF has been the subject of several academic studies. At the Air Force Academy, an EKF was designed for an orbital determination scenario and the filter's stability was explored over a wide range of initial inputs and models. The same initial EKF orbit model and scenario is explored in this paper. While the Academy's effort focused on the stability of the EKF on the initial orbit model, this paper expands from this to explore possible methods of optimizing the EKF results by removing linearizing assumptions made during the initial nonlinear reduction to a linear system.

Linearizing approximations made during the creation of the EKF algorithm are replaced with numerical approximations that allow for nonlinear perturbation effects. The EKF algorithm attempts to minimize the difference in predicted and observed values at each ground site observation. To explore the effectiveness of removing linearization in the algorithm, the root mean square error in each site observation in position (**R**) and velocity (**V**) is explored over a pass of the Mahe Island ground station. Fifteen observations every twenty seconds are made during that pass. However, the true prediction power of the EKF comes from not only accurately predicting the next observation at the same ground station, but also accurately predicting an observation at another station over some large time. The idea of model refinement towards better predictions of the true orbit is similar to the theoretical mathematical model shown later in Fig. 2. The effects of initial model errors and the addition of perturbations to the model are also explored to determine the effectiveness of removing linearizations. With a better understanding of the issues involved in orbital determination and possible ways to improve these results, conclusions can be developed on the linearizing nature of the Extended Kalman Filter.

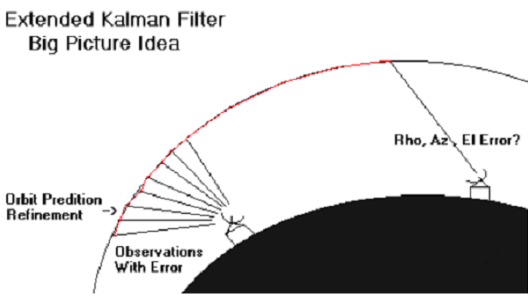

Fig.1 Big picture model of Orbital Detremination

4. Initial Model Assumptions

The Extended Kalman Filter can be a very accurate model for orbital prediction; however, there are several assumptions that go into the design of the filter. The algorithm itself uses a basic model for satellite motion based on the two-body equation of motion. Several perturbation effects are added to this model in this paper, even though there is no way to fully capture all of the characteristics of a satellite's motion. The very nature of the statistical averaging in the algorithm also makes the assumption that none of the input data into the system are completely accurate once any error is specified in the input. As the filter attempts to further linearize the motion of a satellite over time, a first or second order Taylor series is used. This approximation makes the assumption that only one or two terms of a Taylor series are a good approximation. These assumptions help to linearize the nonlinear problem of orbital motion. The purpose of this paper is to remove some of the linearization modeling assumptions in the Extended Kalman Filter.

5. Generalized Math Technique

The Extended Kalman Filter algorithm is made of two primary pieces, a prediction component and a correction component. Both of these components help to create an active algorithm that will statistically compensate a model of an orbit to minimize the observed and predicted position values and velocity state values. The EKF requires several initial inputs in order to work properly. These inputs include: the latitude, longitude, and altitude of the ground site; the individual radar site biases; the initial error covariance matrix (matrix Pi); the error expected from the radar site data (vector R_{error}); the estimated system dynamic modeling error (matrix *Qi*); and the number and type of perturbation effects to consider. The following perturbations are explored: J2, J3, J4, drag, Sun, and Moon (the satellite's ballistic coefficient (BC) will be given to calculate the drag perturbation). In this project, either all or none of the possible perturbation effects are considered. The initial *Pi* matrix, R_{error} vector, radar biases, and ground site data are given for this problem and will remain constant. The initial *Qi* is either zero or (This is the same modeling error estimation assumed from the original Lyapunov Stability analysis.) The nature of these inputs are further explored throughout this paper. The *Qi*, R_{error}, and *Pi* quantities are shown below in equations (1)–(3). The six-by-six nature allows for the position and velocity in each inertial axis to be represented.

Additionally, the initial two-body state model is shown in equations (4) and (5). With these inputs and the initial model established, the EKF is posed to begin its stochastic estimation process.

$$Q_i = \begin{bmatrix} R\hat{i} & 0 & 0 & 0 & 0 & 0 \\ 0 & R\hat{j} & 0 & 0 & 0 & 0 \\ 0 & 0 & R\hat{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & V\hat{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & V\hat{j} & 0 \\ 0 & 0 & 0 & 0 & 0 & V\hat{k} \end{bmatrix} \quad (1)$$

$$R_{error} = \begin{bmatrix} \sigma\hat{i} \\ \sigma\hat{j} \\ \sigma\hat{k} \end{bmatrix} \quad (2)$$

$$P_i = \begin{bmatrix} R\hat{i} & 0 & 0 & 0 & 0 & 0 \\ 0 & R\hat{j} & 0 & 0 & 0 & 0 \\ 0 & 0 & R\hat{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & V\hat{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & V\hat{j} & 0 \\ 0 & 0 & 0 & 0 & 0 & V\hat{k} \end{bmatrix} \quad (3)$$

6. Prediction Process

The Extended Kalman Filter's primary purpose is to establish very accurate predictions of an orbit over time knowing the error that is expected in the model and the actual measurements [1, 2]. The satellite's motion is initially linearized to either a first or second order Taylor series transition matrix. This state transition matrix allows for the prediction of the states of position (**R**) and velocity (**V**) at some future time (Δt)The state transition matrix is based on the rate of change in **V** verses the rate of change of the **R** and is defined as the *F* matrix. Both the state transition matrix (ϕ)and the *F* matrix are shown in equations (6) and (7) respectively.

$$X = \dot{R} \quad (4)$$

$$\dot{X} = \dot{V} = -\frac{\mu}{R^2}\overline{R} + \overline{A}_p \tag{5}$$

$$\phi = e^{-F(t)} = I + F\Delta t + F^2 \frac{\Delta t^2}{2!} \tag{6}$$

$$F = \frac{\partial \dot{\hat{X}}}{\partial \hat{X}} \tag{7}$$

The Cowell method of orbit propagation is used to update the states and the error covariance matrix (*P*). The Cowell method of orbit determination uses the fourth order Runge-Kutta (RK4) approximation method to solve the differential equation for the new states (**X**) at some time (t_n) This differential equation can be seen below in equation (8), where variables with bars over the top are the predicted values and variables with hats are the corrected estimations. The RK4 method propagates the states over 100 time steps between every 20-second observation gap and between the long gap in time to the Thule site. Error covariance of the new states will be predicted from the state transition and the estimated error covariance as shown in equation (9). This idea can be seen in Fig. 2 showing the mathematical process of predicting and correcting the states end error over time.

$$\overline{X}(t_n) = \int_{t_{n-1}}^{t_n} \dot{\hat{X}} dt + \hat{X}(t_{n-1}) \tag{8}$$

$$\overline{P}(t_n) = \phi(t_n, t_{n-1})\hat{P}(t_{n-1})\phi^T(t_n, t_{n-1}) \tag{9}$$

From estimated values, new predicted orbital values can be found, but these values are of little use for further propagation if they do not match the actual observations.

This is where the correction process of the EKF identifies its usefulness. The first step in correcting the system is to establish the error between predicted values and actual observations. This is done through the *H* matrix as shown below in equation (10). Because it is known that the radar site data contains errors as well as the predictions, a statistical analysis process will be needed to correct the predictions and create new estimations of the states. The next step in the process is the development of the Kalman Gain matrix (*K*). The *K* matrix uses the statistical weighed average of the predicted error covariance and the known radar site errors to establish a gain that will seek to minimize the diagonals of the estimated error covariance matrix. The *K* matrix can be seen in equation (11). With this data, estimations of the states and the error covariance can be created from a correction of the original predictions and the actual site information. These correction equations can be seen below in equations (12) and (13), where the [Z-gX] term is the difference in the observed states versus the predicted states.

$$H(\overline{X}(t_n)) = \frac{\partial g(\overline{X}(t_n))}{\partial g\overline{X}(t_n)} \approx \frac{\Delta Observations}{\Delta \overline{X}} \tag{10}$$

$$K(t_n) = P(t_n)H^T\left[HP(t_n)H^T + R_{error}\right]^{-1} \tag{11}$$

$$\hat{X}(t_n) = \overline{X}(t_n) + K\left[Z - g\overline{X}(t_n)\right] \tag{12}$$

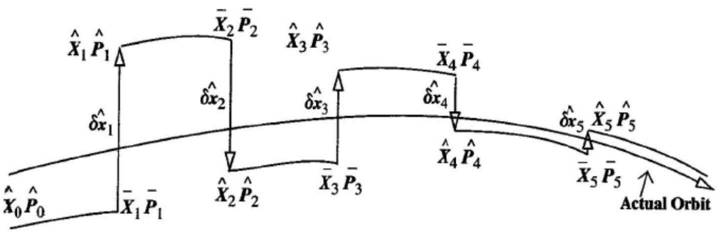

FIG. 2. Correction Process.

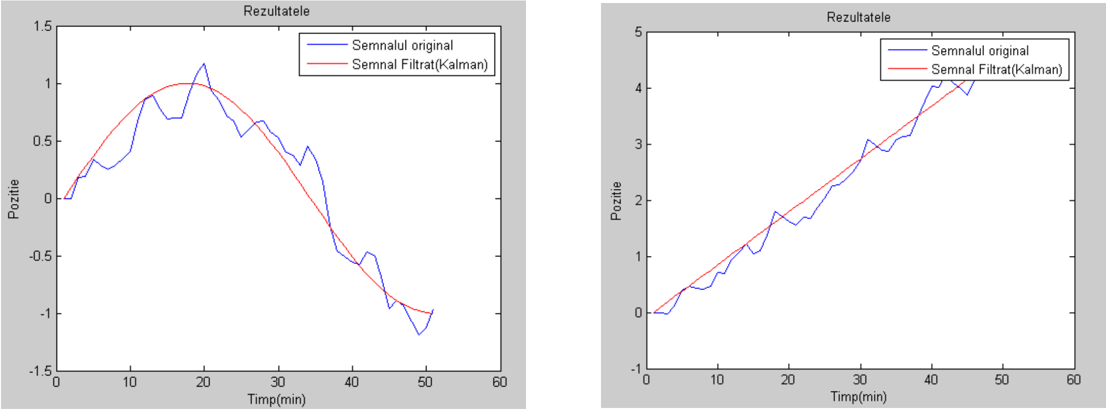

Fig.3. Correction example

Literature

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