ON INVARIANCE OF DESCRIPTION OF OSCILLATORY SYSTEMS IN FIVE-DIMENSIONAL REPRESENTATION

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1. GENERAL CONSIDERATIONS

The interest to fifth-dimensional description of physical processes has arisen in the beginning of the twentieth century in connection with occurrence of article of Th. Kaluza [1] and then A. Einstein basic researches [2-5]. The purpose of fifth-dimensional representation consists in creating the theory, which describe from same positions different fields. The last will permit generalizing exposition of phenomena and to find new regularities. In exposition of physical processes the special place occupies the concept of the metric. It is possible to say that the metric is a totality of coordinates, in which the physical processes are developing, i.e. it is the basic tool of exposition of processes. Really, the overwhelming majority of the physical equations contain functions of spatial and temporal coordinates. The concept of measuring is closely linked to concept of coordinate. Along to spatial coordinates are puts a units of length, and to time coordinate – the units of time and etc. Units of length and time represent the standards. The oscillatory systems serve as the standards of time, and of length. It is enough to recollect that as the standard of time - second, is accepted certain number of periods of atomic oscillation cesium (Cs), and as the standard of length - certain number of wavelengths of atomic radiation krypton (Kr). Otherwise, the temporary and spatial physical measurements are the operations of comparison with characteristics of oscillatory systems: with period T and with distance L between two points having identical phase (for example, maximum of the amplitude). From mentioned becomes clear the interest to fifth-dimensional representation of oscillatory systems and which parameter in this case will serve as the fifth measuring.

In works, quoted above, concept of the fifth-dimensional continuum, represent a generalization of the four-dimensional space-time with addition of the fifth coordinate. In conformity with quoted sources, the fifth dimension has cylindrical property, which consists in following. If a four-dimensional interval dx^{μ} is displaced along the fifth coordinate on some segment ξ^{μ}, this interval will have the same value, as an unbiased. It means that the vector of displacement ξ_{α} exists, which translates the space-time metric in himself. Physically it can be interpreted so: if, all that us surrounds, to displace simultaneously in a direction of the fifth dimension, the final result of such action cannot be observed. This is one of expressions of the principle of a relativity, in conformity with which it is impossible to establish absolute movement in a space

Except temporary and spatial variables, the oscillatory systems are characterized by other parameters (we shall name theirs own), which are connected to spatial - temporary parameters so, that the variation of one, leads to the change of others. It is possible to consider the set of mentioned own parameters, as a some additional fifth coordinate, and the change of this set - as a curvature of the space-time continuum, which consists from the corresponding oscillatory systems.

The set of own parameters of oscillatory system may be considered as a coordinate under condition, if it will form invariant together with spatial and temporary coordinates. Example of invariant in three-dimensional space is the length of the stick, which remains constant when the stick rotates, though the projections to each of coordinates change.

The oscillatory systems irrespective of their nature are submit to the same common principles that allows extending outcomes obtained on one system, on others. The concept about of fifth-dimensional representation of oscillatory systems having various natures is considered below. Such generalization allows to show a generality of the offered approach and to reveal physical sense of the fifth measuring

2. THE ILLUSTRATION OF IDEA WITH THE HELP OF MECHANICAL MODELS

Let us to explain this idea with the help of mechanical system - by the pendulum. The problem can be formulated such as, it is necessary to find the formula, which connects the period and the distance between two maximums with the own parameters of pendulum, under the condition - the variation of any of these characteristics must not change found expression.

The movement of a pendulum is characterized by the equation, which describes a wide class of oscillatory systems:

$$m\frac{d^2l}{dt^2}+kl=0\,, \qquad (1)$$

where: $k = -F/l$ - factor of elasticity, characterizes the resistance of system to a deviation

$$l=\sqrt{x^2+y^2+z^2}$$

from the balance state;

$$m=\frac{F}{v/t}=\frac{F}{a}\text{ -}$$

inertness, which characterizes the resistance, opposed by the oscillatory system to an acceleration, otherwise, at attempt to transfer it into an other reference system.

The inertness is identified with mass for mechanical oscillatory systems, however it is not so always. For example, light has inertness but has not mass. At an electromagnetic field the presence of inertness follows from existence of electromagnetic oscillations, which are possible, if we have two components - inert and elastic.

The solution of the equation (1) represents harmonic function $l = L\cos\omega t$, which satisfies (1) under the condition

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{F_a}{mL}} \qquad (2)$$

where L - amplitude of a deviation and F_a - amplitude of return force.

If $\omega = 2\pi/T$, and the complete energy of oscillatory system is equal to the potential energy in the deviation maximum position:

$$W=\frac{LF_a}{2}\,, \qquad (3)$$

then it is possible to rewrite(2) as

$$L^2-\frac{1}{2\pi}\frac{W}{m}T^2=0\,. \qquad (4)$$

The coefficient

$$c^2=\frac{W}{m}\,, \qquad (5)$$

expresses the quantity of energy, which corresponds to the unit of inertness of oscillatory system, otherwise, c^2 is a specific energy capacity of the given oscillatory system. On the other hand c^2 connects temporary T and spatial L scales of oscillatory system.

The expression (4) and (5) are correct for any oscillatory systems, containing generalized elastic and inert elements in that sense, in which it was determined above. In such systems the inert element cyclically repeats the same trajectory. The length of this trajectory L can be accepted as scale of length, and the period T - as scale of time (the latter is realized at the clock).

Now, we shall imagine, that, there is the sequence of oscillatory systems, located so, that the energy can be transferred completely from one element to other. For example, it can be a chain of balls, connected among themselves by springs.

If the first ball in such chain will be pushed, he will compress a spring located beside, which in turn, will push the following ball and so on. At the result, a wave will be propagating in this chain. The difference between the wave and the oscillatory process of isolated system, consists in that that the energy from an elastic element does not come back to an inert element from which was received, but is transferring to following. It means, that the transferring of energy gets a spatial orientation, and the oscillatory process is unwrapped in space. In this case, the length of a trajectory of an inert element will be replaced by the distance λ between two neighbour elements inert, which are in an identical phase. And $c=L/T=\lambda/T$ parameter, which connects spatial and temporary variable, receives the sense of velocity of propagation of perturbation, in addition. In case of propagation of a wave in a chain, the exchange by energy between the neighbour inert and elastic elements occurs by the same mode as in isolated oscillatory system, therefore the relation (4), which contains parameter (5), remains correct.

If in a chain of oscillatory systems is included an element, for which the ratio W/m (the specific capacity of energy) differs from other elements, then, according to (4), for him ratio between time scale T and length L will be also different. This will be perceived as “curvature of the spatial-temporary continuum” of the chain of oscillatory elements.

When the balls are very small and are located closely one to another, it is possible to carry out transition to a limit, and a chain may be considered as a continuous. Usually is used the term “continuum”. In this case, the separate elements cannot be identified, therefore it is necessary to operate with distributed per unit of length or per unit of volume inertness and elasticity, instead of the parameters of separate oscillatory elements.

The perturbation, which travels in such medium, gets properties of independent object, not connected with elements of medium. It is enough to recollect the waves on a surface of the water, which create impression of a movement of the all lake, though the water remains on the place.

It is possible to show, that in the case of continuous medium there are expressions such as (4) and (5), but the change of rapport between distributed elastic and inert components will lead to the change of ratio between spatial and temporary scales. It means that this change will be perceived as curvature of the metric of this continuous medium.

3. THE APPLICATION OF IDEA FOR ELECTROMAGNETIC WAVES

The electromagnetic waves have a special role in a nature, on their properties the theory of relativity is based, besides they have the same nature as the elementary particles. So long as all physical objects without exception are subordinated to the relativity theory, the results received for electromagnetic model, concerning the space-time continuum, can be generalized on all that us surrounds.

The relation between coordinates, which characterizes the metric of world, is determined by the interval

$$S = \sqrt{l^2 - c^2 t^2},$$

Where c is velocity of electromagnetic waves in vacuum, l and t - segments of length and time. The concept of an interval appears from Lorentz transformations, which are founded on idea of invariance in relation to a frame of reference of a spherical light wave or, otherwise, of light complex [1], which is meaning radiation contained between two spherical surfaces with common center in the moment t. In particular, one of these surfaces can be tightened in a point. The light complex can be considered as some vibrating system

In an above deduction we did not impose any conditions on a nature of oscillatory systems, except a condition, that they contain inert and elastic components. Hence, in case of electromagnetic waves, also will be valid the reasoning about existence of parameter connecting inert and elastic components. His change under the influence of any external factors will entail the change of rapport between scales of length and time, and consequently the change of a space-time metric. This parameter, having in addition the sense of specific energy capacity, can be considered as fifth coordinate. The cylindrical condition, formulated in the beginning, will be carried out if the aspect of expression for an interval will be kept in case of change of specific energy, it means in case of moving in a direction of the fifth coordinate.

We shall consider a deduction, similar done for mechanical oscillatory system, with reference to electromagnetic waves. In case of the electromagnetic waves, it is necessary to replace elastic and inert components with electrical and magnetic fields accordingly. The relation between them is defined by the Maxwell’s equations. In the beginning, we shall be limited by a case one-dimensional plane polarized wave, propagating in direction of axis x. In this case the Maxwell’s equations will look like:

$$\mu \frac{\partial H_Y}{\partial t} = \frac{\partial E_Z}{\partial x} \qquad (6)$$

$$\varepsilon \frac{\partial E_Z}{\partial t} = \frac{\partial H_Y}{\partial x}. \qquad (7)$$

Here H_Y and E_Z - intensity of magnetic and electrical fields; μ and ε - magnetic permeability and dielectric constant, accordingly.

By dividing (6) on $\sqrt{\mu}$, and (7) - on $\sqrt{\varepsilon}$, after subtraction of the received equations, we shall obtain:

$$\frac{\partial\left(\sqrt{\mu}H_Y - \sqrt{\varepsilon}E_Z\right)}{\partial t} + \frac{1}{\sqrt{\varepsilon\mu}} \frac{\partial\left(\sqrt{\mu}H_Y - \sqrt{\varepsilon}E_Z\right)}{\partial x} = 0 . \qquad (8)$$

Here and further we consider parameters ε and μ locally constant, it means we neglect of their change on distance of wavelength and during one period.

(8) describes propagation of a wave along an axis x with velocity $c = 1/\sqrt{\varepsilon\mu}$. We suppose, that this wave is described by some function of coordinate x and time t:

$$f(x - ct) = \sqrt{\mu}H_Y - \sqrt{\varepsilon}E_Z,$$

From here for a components of a field:

$$H_Y = \frac{1}{2\sqrt{\mu}} f(x - ct),$$

$$E_Z = -\frac{1}{2\sqrt{\varepsilon}} f(x - ct).$$

On the other hand, after multiplication (6) on H_Y, and (7) on E_Z and their addition, we shall obtain:

$$\mu\frac{\partial}{\partial t}\frac{H_Y^2}{2}+\varepsilon\frac{\partial}{\partial t}\frac{E_Z^2}{2}-\frac{\partial}{\partial x}(E_Z H_Y)=0\,. \quad (9)$$

If to consider parameters ε and μ locally constant, as well as before, the equation (9) can be copied as:

$$\frac{\partial}{\partial t}\left(\mu\frac{H_Y^2}{2}+\varepsilon\frac{E_Z^2}{2}\right)-\frac{\partial}{\partial x}\left(E_Z H_Y\right)=0\,.$$

This identity is a condition of existence of the curvilinear integral:

$$U=\oint\left[\left(\mu\frac{H_Y^2}{2}+\varepsilon\frac{E_Z^2}{2}\right)dx-\left(E_Z H_Y\right)dt\right]. \quad (10)$$

The expression (10) describes behaviour of light complex, or of the oscillatory system, which is interest for us. Really, integral

$$W=\int_{x_1}^{x_2}\left(\mu\frac{H_Y^2}{2}+\varepsilon\frac{E_Z^2}{2}\right)dx \quad (11)$$

represents the complete energy of an electromagnetic field in the moment of time t_1 in a layer, being between the planes x_1 and x_2 in volume $V=SL$ (figure 1), at the condition, that the area S is equal to unit. Here $L=x_2 - x_1$

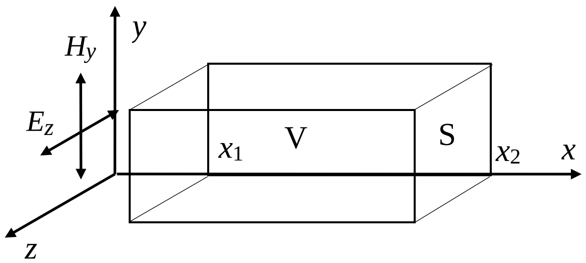

Figure. 1. The considered light complex is in volume *V*.

The expression $P=(E_Z H_Y)$ - is the Poynting vector or the density of an energy flow. Then

$$Q=\int_{t_2}^{t_1}\left(E_Z H_Y\right)dt$$

- is the energy, passing through the unitary area S with coordinate x_2 during $T = t_2 - t_1$. Thus, if we shall choose in quality of the oscillatory system the light complex, contained in volume $V = SL$ and which leaves V through a plane S during T, then the expression (10) characterizes this light complex.

It is possible to copy the formula (10) on the basis of the theorem about average:

$$\left[\mu\frac{H_Y^2}{2}+\varepsilon\frac{E_Z^2}{2}\right]L=\left[E_Z H_Y\right]T\,, \quad (12)$$

Where:

$$\left[\mu\frac{H_Y^2}{2}+\varepsilon\frac{E_Z^2}{2}\right]=[w] \quad (13)$$

- is a some average value of energy density of light complex, contained in volume V in the moment of the time t_1

$$\left[E_Z H_Y\right]=[P] \quad (14)$$

- is a some average value of density of the energy flow of light complex, which passes through S during time T. With the account (12) and (13), (14) gets a aspect:

$$[w]L=[P]T\ . \quad (15)$$

Density of the energy flow P is connected with density of a impulse flow g by expression:

$$P=cg, \quad (16)$$

On the other hand, from the definition of the g, follows, that, the complete impulse of a light complex contained in V:

$$p\ =\ [g]TS\,. \quad (17)$$

Taking into consideration (16) and (17), and that by definition $c = L/T$, the expression (15) may be copied:

$$[w]L=\frac{p}{TS}\,, \quad (18)$$

but between a complete impulse p and inertness m , which has oscillatory system, exists relation:

$$p=mc=[\rho]Vc=[\rho]SLc=[\rho]SL^2/T \quad (19)$$

where ρ - the average density of inertness of a light complex.

Expression (19) - allows clarifying a difference between the inertness and the mass of oscillatory system. As it was marked, the electromagnetic complex has not mass, however, he has the determined inertness, which is equal to the ratio between impulse and velocity of this complex.

In particular, when the light complex, which travels with the velocity of light, is absorbed, thus he is translated in laboratory system.

By taking into account the formula (19), the expression (18) will be finally copied:

$$L^2 - \frac{[w]}{[\rho]} T^2 = 0. \qquad (20)$$

If to divide the numerator and denominator of expression (4) to the volume, this expression will become identical to (20), with accuracy up to constant multiplier $1/2\pi$. Thus, for a light complex is also valid the expression which characterizes oscillatory systems and which connects scales of length and of time.

The above mentioned conclusion can be simply generalized for a case of three spatial coordinates and the circular polarization. For this purpose the three-dimensional equation of continuity is used for volumetric density of the energy instead of expression (11):

$$\frac{\partial w}{\partial t} + div\boldsymbol{P} = 0,$$

or, in the tensor form used in quoted works:

$$\frac{\mathbf{T}_{k4}}{X_k} = 0,$$

where $\mathbf{T}_{k4}$ - the energy-momentum tensor, k = 1,2,3,4. In this case spatial scale becomes three-dimensional:

$$L = \sqrt{L_X^2 + L_Y^2 + L_Z^2},$$

where L_X, L_Y, L_Z - are projections of L on the spatial axes.

4. CONCLUSIONS

We have shown, that all oscillatory systems, including electromagnetic oscillations have one universal property, namely, are characterized by expression, similar to an interval in the special theory of relativity. However, in our case, factor connecting spatial and temporary components, gets sense of specific energy capacity of the space-time continuum. This parameter carries out a role of the fifth dimension. The cylindrical property is observed, because the form of expression for the interval S remains invariant when the energy capacity of the space-time continuum changes.

Taking into account that the oscillatory systems are applied as the standards of length and time, it is possible to make a conclusion, that the found interrelation between spatial-temporary dimensions and the specific energy capacity is universal.

As far as the offered hypothesis corresponds to the experimental facts? It is known that the electromagnetic radiation, getting into a gravitational field, changes its frequency [6]. In turn, the change of frequency of light ν is connected with the change of its energy W by a formula Plank $W = h\nu$. The energy W can be attributed to a fixed light complex with volume V. Then change of this energy, which is connected with the change of frequency, will cause the same change of energy density $w = W/V$. Further, by virtue of the equation (20), relation between spatial and temporary scales will be changed, hence the fact, which is named "curvature of the space-time continuum" will take place.

Remark. The velocity of light changes also in a case, when the light penetrates medium for which the refraction coefficient differs from unit. However, the mechanism of change of velocity in this case is the other than in a gravitational field. The passage of light through substance can be presented as a repeated absorption and radiation. In this case the frequency of light does not change, while in a gravitational field the frequency of light changes. Therefore the delay of the velocity of light in substance in comparison with vacuum, cannot be identified with curvature of the space-time continuum.

Thus, it is possible to make the following conclusion. The curvature of the space-time continuum, caused by a gravitational field, consists in the change of the energy capacity of the vacuum.

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