THE ROLE OF THE ALGEBRAIC STRUCTURES IN COMPUTERS' DESIGN

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For the numbers' representation, symbols called digits are currently used. The positional systems have strict representation rules meaning that the order of the digits is decisive, being easier to interpret and put it in practice, being generalized in counting. Their discovery is available because of the implementing of the "0" figure whose historic importance could be compared only with the technical discovery of the wheel, more precisely of the circle.

It is well known the fact that the positional numeration systems have the following important components: the numeration base, digits and digit's rank.

The numeration base (B) is a standard that establishes the relation between the absolute value of the number and its representation. For a bigger base we will have a simplified number expression meaning a shorter one.

The rank of a digit means its weight in one number's value. The bigger is the digit's rank the more significant it becomes.

The general rule of representing a number in a base is the following:

$$N_B = c_r\, c_{r-1} \ldots c_2\, c_1\, c_0,\, c_{-1}\, c_{-2} \ldots$$

The value of the number can be found by summing the products between the digit and base powered to the digit's rank. This number's representation can be associated with a polynomial expression where the digits are the coefficients the powers of the base are multiplied with, respectively:

$$N_B = c_r \cdot B^r + c_{r-1} \cdot B^{r-1} + \ldots$$
$$+ c_2 \cdot B^2 + c_1 \cdot B + c_0 + \frac{c_{-1}}{B} + \frac{c_{-2}}{B^2} + \ldots$$

The universal system of numeration, frequently used, is the decimal system with base B = 10 whose digits are:

$$B_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

Due to the appearance of the electronic computers and the Boolean Algebra, other systems of numeration have developed, for example the binary system with $B_2=\{0, 1\}$.

Successive groups related to the digit of superior rank make the conversion of a decimal number to a binary one. The base B, keeping the rest as a digit with the units' rank, divides the given number and the quotient follows the some logic of grouping for the digits having a higher rank.

The implementation of the binary system was facilitated by the computer's existence. In their electronic circuits, the "*0*" and "*1*" digits stand for two different electrical levels respectively 0 mans switch off and 1 means switch on. By analog with this, if base was used, 10 different levels should be used, which could be difficult to achieve in technological terms.

The Mathematical Logic, namely the bivalent Logic can be applied not only Mathematical Linguistics such as automatic translation and the code's theory, algorithm's theory but also in programming theory [2].

The multitude of statements can be divided into two classes but this isn't the only possibility. The polyvalent Algebras (trivalent, pentavalent) play the same role in the polyvalent Logic similarly to the one played in the Boolean algebra by the classical statements calculation.

In the logic that has three values, respectively true *T*, false *F*, possible *P*, we must apply the excluded quart principle.

There must be considered four values logic respectively: necessary true T_n, contingent true T_c, necessary false F_n, contingent false F_c in which the excluded quint principle finds its application [1].

If in the trivalent Logic, we consider statements that are not definitely true (contingent), we obtain the pentavalent Logic and a multitude is implemented:

$$L_5 = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}.$$

The excluded six principle states that all the five possibilities stand for the statement's multitude and the contradiction principle states that these five possibilities are excluded two by two.

Of course, next we can move on to the *n* values Logic respectively the multitude:

$$L_n = \left\{ 0, \frac{1}{n-1}, \frac{2}{n-1}, \ldots, \frac{n-2}{n-1}, 1 \right\}$$

This case brings into discussion the philosophical matter of sense.

Coming back to the bivalent Logic, we can consider the Boolean function or the *n* variables binary function as being a function on the Cartesian product

$$B2 \; x \; B_2 \; x \; \ldots \; x \; B_2$$

which has values in B2.

Taking into account the fact the definition domain of a B_2^n Boolean function has 2^n elements the functions multitude will have 2^{2^n} functions. An important matter related to the Boolean functions representation is to find a simplified form for its expressions.

The Boolean function physical realization is facilitated by the study of the circuits with contacts.

The operations disjunction (+) and conjunction (·) are interpreted by serial and parallel convection of the dipoles.

In the electrical equipments, complex logical circuits are used, such as Keyboard, printer and processor, the last being considered one of the most important circuits.

From the Veitch-Karnaugh diagrams results simplified Boolean expressions in their only possible representation.

For the case of the processor, the Boolean expression including the transport digit is:

$$s = \bar{a}\bar{b}t + \bar{a}b\bar{t} + a\bar{b}\bar{t} + abt,$$

$$t' = ab + at + bt,$$

Where *a*, *b* are binary digits to sum up, *t* is the binary digit of transport, *s* is a sum digit and t' is the new digit of transport [3]. The diagrams from above represent, in fact, the reordering of the Euler-Venn diagrams as an array in which the minterms are emphasized by adjunctive squares, being about canonic disjunctive forms [3]. Gr. C. Moisil together with an engineer, Gh. Ioanin, considered the relays with multi-positional contacts with three respectively five positions (in a word with intermediate positions) and this thing permitted the technical achievement of some polyvalent Logics [1], and some algebraic studies that tackle the proper numeration bases. The study of these Logics can be reconsidered by the possibility of interpreting the logical values as probabilities or, more deeply, by attaching a supplementary type of statements the ones without sense.

By analogy with this, in order to study the principles of Logic, we must begin with the idea of a chain implemented for the first time by the calculations of probabilities which runs the philosophy of nature not only under the aspect of determination but also under its statistical form.

It is to be noticed that the trivalent Logic $L_3 = \left\{ 0, \frac{1}{2}, 1 \right\}$ is isomorphic with the ring of the categories of rests of integers modulo 3.

The procedure of converting a decimal number to base 3 (in MathCAD language) [4], will be presented below:

$$\mathrm{tri}(N) := \left| \begin{array}{l} u \leftarrow \mathrm{identity}(1) \\ u \cdot N \quad \text{if } N \le 2 \\ \mathrm{stack}\left(u \cdot \mathrm{mod}(N,3), \mathrm{tri}\left(\mathrm{floor}\left(\frac{N}{3}\right)\right)\right) \quad \text{otherwise} \end{array} \right.$$

$$\mathrm{tri}(7) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad .$$

The result will be read as follows: $7_{10} = 21_3$.

For illustrating the working method into another numerical base, the table of addition in base 3 will be presented as follows:

The addition table in base 3.

	[illegible]		2
	[illegible]		2
	[illegible]		1
		0	
	[illegible]		1
	0	1	

By analogy with this, we can consider a multiplication table and execute several operations in the new base. For the computer, the multiplication is a repeated addition and this fact must be taken into consideration.

In conclusion, the binary system is generalized for construction of numerical computers. All the 16 logical functions we referred to are frequently used in programming activity, knowing them being an essential fact [2]. The Keyboard has the role to automatically convert the information to a binary numeric code by a programmed system of cables that have a Codifying role.The Central unit, by its processor takes these binary numbers and by the help of Summatory executes the addition. Let's specify that the addition is the only cabled operation according to the above-presented logic, the multiplication being operated by repeated additions.

The result we get is showed on the screen or at the Keyboard after the Keyboard changed it into a decimal. Apparently, this is not a convenient way of working and this is the reason the interfaces are looked for, in order to facilitate a friendly presentation. That's why, the lack of a MathCAD language from the common activity leads to an inferior position. Without effective Knowing of the language, the simple using of Math command button offers the possibility of language working.

The nine command buttons are [4]: Calculator Toolbar, Graph, Vector and Matrix, Evaluation, Calculus, Boolean, Programming, Greek Symbol, Symbolic Keyword.

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