A NEW APPROACH TO KINEMATICS OF 1DOF PLANETARY TRANSMISSIONS

R. Velicu, C. A. Bozan
Transilvania University of Braşov

INTRODUCTION

Planetary mechanisms have a large use on the construction of motor vehicles, machine tools and industrial robots transmissions. Among planetary transmissions for motor vehicles an important place is dedicated to planetary gearboxes, from which the most common solutions are the one consisted of two planetary units, called two-planetary group gear boxes. A planetary unit is defined [1] as a 2 DOF gear mechanisms with three central elements: two gears (central elements 1 and 4) with the same rotational axis (central axis of the planetary unit; one carrier H, with the same rotational axis. Fig. 1 presents a structural scheme (a), the symbolical structural scheme (b) and the generalized symbolical structural scheme (c) of a planetary unit. The generalized central elements of the two planetary units are **m**, **n** and **p**.

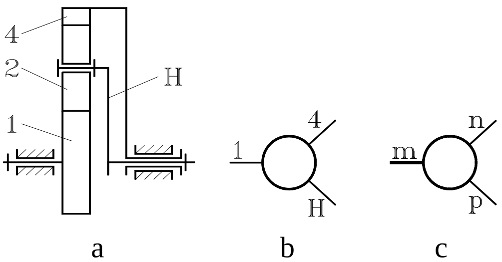

Figure 1. Planetary unit.

A two-planetary group is consisted of two planetary units, with two links between their central elements. Fig. 2 presents a concrete structural scheme (a), the corresponding symbolical structural scheme (b) and the generalized symbolical structural scheme (c) of a two-planetary group. The generalized central elements of the two planetary units are **m1**, **n1**, **p1**, respectively **m2**, **n2**, **p2**, the single elements of the two-planetary group are **a** and **c**, and the double elements **B** and **D**

The 1DOF planetary transmission is obtained from the 2DOF planetary mechanism, by blocking a central element, which will be noted as *fix*. The input central element will be noted as *inp*, and the output central element will be noted as *out*.

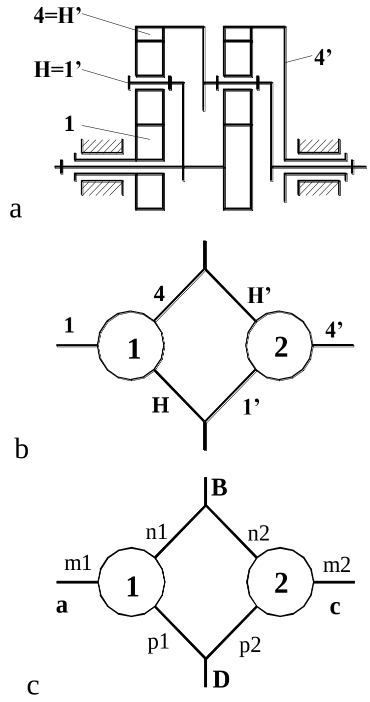

Figure 2. Two-planetary group.

1. KINEMATICS

1.1. Transmission ratios relationships

Kinematics of a mechanism is mainly dealing with the dependencies between the movements of the mechanism's elements. In the case of planetary mechanisms, most important are the dependencies of the movements of the central elements, called transmission ratios.

Most of planetary transmission references [1, 3, 4, 6] use as a basic element the interior transmission ratio i_0. This is the transmission ratio of the planetary unit with fixed carrier, calculated as

$$i_0 = i_{0(14)} = i_{14}^H = \frac{\omega_{1H}}{\omega_{4H}} = \frac{n_{1H}}{n_{4H}} = = \frac{n_1 - n_H}{n_4 - n_H} = \left(\frac{n_{1H}}{n_{4H}}\right)_{H\equiv 0} \tag{1}$$

depending on the number of teeth of the gears consisting the planetary unit.

In order to simplify the analyze of kinematics of planetary transmissions (like planetary units, two or three-planetary groups), the literature [4, 6] is giving relationships for the transmission ratios, depending on the interior transmission ratios of the consisting planetary units.

The main inconvenience in writing these relationships is their large number, which makes difficult their use in software development. As an example, analyze of each of the three generalized symbolical scheme of three-planetary group, needs a set of 60 generalized relationships.

The speeds of the central elements of any 1DOF planetary mechanism are depending on the speed of the input element ω_{inp} and a transmission ratio.

Starting from the common calculus relation of a transmission ratio

$$i_{inp\,x}^{fix} = \frac{\omega_{inp\,fix}}{\omega_{x\,fix}} = \frac{\omega_{inp}}{\omega_x}, \tag{2}$$

the angular speed of any central element x is

$$\omega_x = \frac{\omega_{inp}}{i_{inp\,x}^{fix}}. \tag{3}$$

For the particular cases $x \equiv y$, $y \equiv z$ and $x \equiv z$ the transmission ratio i_{xy}^{z} becomes:

- $i_{xy}^{z} = i_{xx}^{z} = \omega_{xz} / \omega_{xz} = 1$, for $x \equiv y$;
- $i_{xy}^{z} = i_{xy}^{y} = \omega_{xy} / \omega_{yy} = +\infty$, for $y \equiv z$;
- $i_{xy}^{z} = i_{xy}^{z} = \omega_{xx} / \omega_{yx} = 0$, for $x \equiv z$.

In order to avoid the transmission ratio $i_{xy}^{y} = +\infty$, in the calculation of the angular speeds ω_x and the ratio to the input element angular speeds ω_{inp}, ω_x/ω_{inp}, the following calculus relations can be used:

$$\omega_x = \omega_{inp}\, i_{x\,inp}^{fix}; \tag{4}$$

$$\frac{\omega_x}{\omega_{inp}} = \omega_{xinp} = i_{x\,inp}^{fix}, \tag{5}$$

the input *inp* and the blocked *fix* elements being never identical.

This paper is proposing a method of solving the kinematics of any planetary transmission by using a very small number of relationships.

Due to the fact that the relative motions between any three central elements x, y and z do not depend on any other element, between their relative speeds, the following relations can be written $\omega_{xy} = \omega_{xz} - \omega_{yz}$ and $\omega_{xy} = -\omega_{yx}$.

For any complex planetary mechanism the following relations can be written:

$$i_{xy}^{z} = \frac{\omega_{xz}}{\omega_{yz}} = \frac{\omega_{xy} - \omega_{zy}}{-\omega_{zy}} = 1 - \frac{\omega_{xy}}{\omega_{zy}} = 1 - i_{xz}^{y}; \tag{6}$$

$$i_{xy}^{z} = \frac{\omega_{xz}}{\omega_{yz}} = \frac{1}{\dfrac{\omega_{yz}}{\omega_{xz}}} = \frac{1}{i_{yx}^{z}}; \tag{7}$$

$$i_{xy}^{z} = \frac{\omega_{xz}}{\omega_{yz}} = \frac{\omega_{xz}}{\omega_{uz}}\frac{\omega_{uz}}{\omega_{yz}} = i_{xu}^{z}\, i_{uy}^{z}. \tag{8}$$

Relationships (6), (7) and (8) are expressing dependencies between the gear ratios of any complex planetary mechanism. The methodology to determine the analytical expressions of any transmission ratio, based on relationships (6), (7) and (8) is purely deductive.

Relations (6) and (7) are enough to determine the possible transmission ratios of a planetary unit, depending on their interior transmission ratio i_0. As an example, the transmission ratios of a planetary unit can be determined as follows (see fig. 1):

$$\begin{aligned}
& i_{14}^{H} = i_0; \\
& i_{1H}^{4} \xrightarrow{with\ rel.(6)} 1 - i_{14}^{H} = 1 - i_0; \\
& i_{41}^{H} \xrightarrow{(7)} \frac{1}{i_{14}^{H}} = \frac{1}{i_0}; \\
& i_{4H}^{1} \xrightarrow{(6)} 1 - i_{41}^{H} \xrightarrow{(7)} 1 - \frac{1}{i_{14}^{H}} = 1 - \frac{1}{i_0}; \\
& i_{H1}^{4} \xrightarrow{(7)} \frac{1}{i_{1H}^{4}} \xrightarrow{(6)} \frac{1}{1 - i_{14}^{H}} = \frac{1}{1 - i_0}; \\
& i_{H4}^{1} \xrightarrow{(7)} \frac{1}{i_{4H}^{1}} \xrightarrow{(6)} \frac{1}{1 - i_{41}^{H}} \longrightarrow \\
& \xrightarrow{(7)} \frac{1}{1 - \dfrac{1}{i_{14}^{H}}} = \frac{1}{1 - \dfrac{1}{i_0}}.
\end{aligned} \tag{9}$$

From relations (9), there can be seen that the steps are pursuing the change of the position of the central element H in i_{xy}^{z} relation, in order to reach the position of the z element. Position is changed in the order x → y → z.

For the case in which the three central elements x, y and z involved in the transmission ratio i_{xy}^{z}, belong to two planetary units, all three relationships (6), (7) and (8) must be used.

As an example, the transmission ratio of the 1DOF two-planetary group presented in fig. 3, is established in the following steps:

$$i_{Dc}^{a} \xrightarrow{(7)} \frac{1}{i_{cD}^{a}} \xrightarrow{(6)} \frac{1}{1-i_{ca}^{D}} \xrightarrow{(8)} \frac{1}{1-i_{cB}^{D}\, i_{Ba}^{D}} = \\ = \frac{1}{1-i_{H'1'}^{4'}\, i_{H4}^{1}} \xrightarrow{(7),(7)} \frac{1}{1-\dfrac{1}{i_{1'H'}^{4'}\, i_{4H}^{1}}} \xrightarrow{(6),(6)} \\ \rightarrow \frac{1}{1-\dfrac{1}{\left(1-i_{1'4'}^{H'}\right)\left(1-i_{41}^{H}\right)}} \xrightarrow{(7)} \\ \rightarrow \frac{1}{1-\dfrac{1}{\left(1-i_{1'4'}^{H'}\right)\left(1-\dfrac{1}{i_{14}^{H}}\right)}} = \frac{1}{1-\dfrac{1}{\left(1-i_{02}\right)\left(1-\dfrac{1}{i_{01}}\right)}}. \tag{10}$$

From relations (10), there can be see that the first steps are changing the position of the common central element of the two units (D), in order to reach the position of element z in i_{xy}^{z} relation (first two steps). Position is changed in the order x $\rightarrow$ y $\rightarrow$ z. Then, by applying relation (8), the expression becomes dependant only of ratios involving central elements of a single planetary unit.

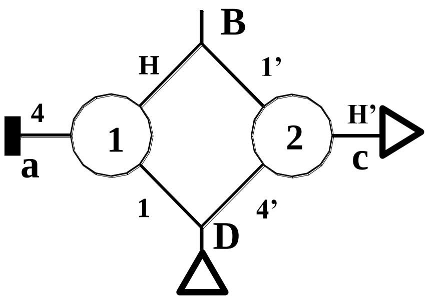

Figure 3. Particular 1DOF two-planetary group.

The last example is one of the most complex transmission ratio of a two-planetary group.

1.2. Software development

The computer aided kinematics analyze must allow the establish of any central element angular speed ω_x. The minimum input data are: structural scheme of the mechanism; the interior transmission ratios of the consisting planetary units.

Calculus of the transmission ratios i_{xy}^{z} is based on using relationships (6), (7) and (8), depending on the already known transmission ratios. Three procedures are used:

Procedure Rule1; {Use of relation $i_{xy}^{z} = 1 - i_{xz}^{y}$ }

```
begin
  if i.def.[x,z,y]=TRUE then
  begin
  i.val[x,y,z]:=1- i.val[x,z,y];
  i.def.[x,y,z]=TRUE;
  end;
end;
```

Procedure Rule2; {Use of relation $i_{xy}^{z} = \dfrac{1}{i_{yx}^{z}}$ }

```
begin
  if i.def.[y,x,z]=TRUE then
  begin
  i.val[x,y,z]:=1/ i.val[y,x,z];
  i.def.[x,y,z]=TRUE;
  end;
end;
```

Procedure Rule3; {Use of relation $i_{xy}^{z} = i_{xu}^{z}\, i_{uy}^{z}$ }

```
begin
  if (i.def.[x,u,z]=TRUE) and
  (i.def.[u,y,z]=TRUE)
  begin
  i.val[x,y,z]:= i.val[x,u,z]*i.val[u,y,z];
  i.def.[x,y,z]=TRUE;
  end;
end;
```

The following notations have been used: i.def. [x, y, z] = TRUE, the transmission ratio i_{xy}^{z} is known (first settings are i.def. [x, y, z] = FALSE for all unknown transmission ratios, i.def. [x, y, z] = TRUE for the known interior transmission ratios); i.val [x, y, z] is the value of the transmission ratio i_{xy}^{z}.

Starting from the known transmission ratios, by repeatedly appealing the above procedures (variables x, y, z and u successively changing) new transmission ratios become known (i.def. [x, y, z] = TRUE) and forward usable. The cycle is finalized when all the transmission ratios i_{xy}^{z} become known.

2. SOFTWARE

A software was developed for the analyze of two-planetary groups. Fig. 6 shows the input panel. Results can be presented as diagrams.

An example for the results of the analyze of the two-planetary group defined in fig. 6 is presented in fig. 7, 8 and 9.

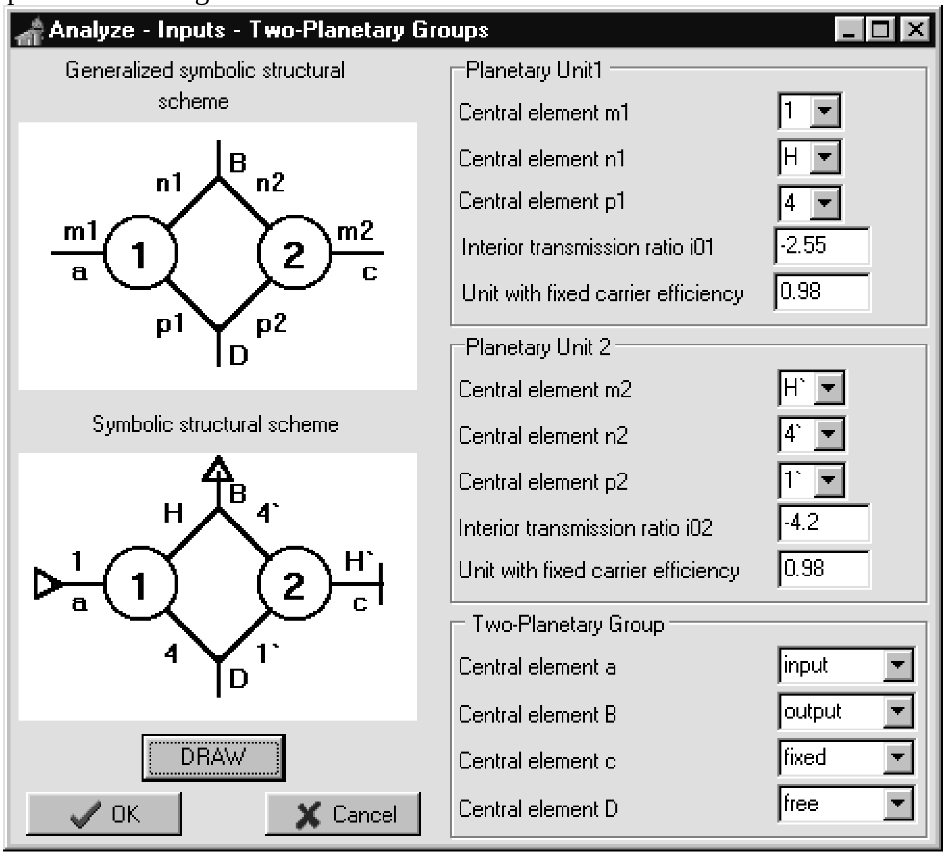

Figure 4. Input data panel.

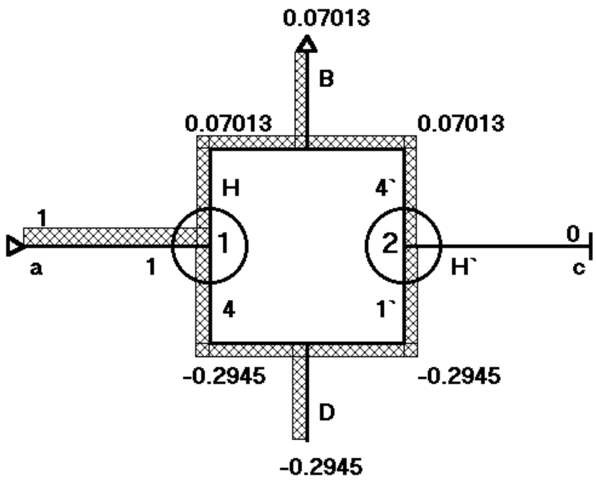

Figure 5. Angular speeds on central elements ω_x/ω_{inp}

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