MATHEMATICAL MODEL FOR THE ESTABLISHMENT OF VARIATION SIZE OF THE MAIN CUTTING FORCE COMPONENT

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INTRODUCTION

The body surfaces of manufacture are much more “*scraggily*” than the multitudes considered in the classic geometry; if these are enlarged more irregularities become visible. A boundary surface, among two materials look very smooth if we look at them from distance, but, as the distance is decreased, very many irregularities are visible.

The fractal multitudes, seemed to be in the beginning just a pure mathematical notion, found very numerous applications in technique.

Fractal (word which drifts from fractus, and which in Latin means the fracture, deriving in sequence from the verb franger ≡ to tear), suggests such multimode.

In his work “The Fractural Geometry of Nature” (1982), the American of polish origin Benoit Mandelbrot argues that such geometrically abstracts fitted frequently with the physical world, better than the curves and smooth surfaces.

1. GENERAL CONSIDERATIONS

The materials with reduced workability are those by-paths categories of materials witch manufacture through cutting tool raises distinguish problems, either the aspect of the cutting tool or from the mechanical application point if view and the energy generated at the time of cutting.

The majority operations that are being accomplished to obtain the machine part used in the engineering of a car (arbores, denticulate wheels, screws, feather, etc) are operations of cutting.

For the control of the cutting tool it must be taken in to consideration the influence which it has on the accuracy of the manufacture taking into account: the peaks usage and dilatation, the method of obtaining the sizes and geometrical forms (on the side or the adjustment of sizes), the machines tool adjustment (the adjustment size of the semi product that is manufactured).

Manufacturing machine parts through adjustment of size, the peaks usage and dilatation from the cutting tool, in time, the machine parts obtained vary in the field of tolerance.

The systematic variation errors determined by heat and usage of the cutting tool, establishes the average size values of manufactured machine parts.

The adding relations for the plastic deformation and friction forces of the cutting tool, in the trirectangular system (*Oyez*) presupposes supplementary calculation necessary for the determination of intermedial sizes (conventional angel of shearing, unitary efforts from the shearing plan and the cutting tools surface of diffuse, exterior angel of friction, the cutting tools plastic deformation value).

The cutting tool influence efficient the process of manufacture through the necessary auxiliary times prerequisite, reshaped or change (in the case of advanced usage or tear), because the duration of a stationary process centre can results in a breakdown of the cutting tool is 6,8% in total time, concurring [2].

In the process of cutting tool the semi product it is very important to obtain a end product in the tolerance field specified (prescribed by the designer).

This mathematical model takes in to consideration: the depth of the cutting tool, the advance and the Brinnel hardness as well as several values of correctness.

The most used experimental relations that help to determine the main force of the cutting tool, in the process of turning, drilling and milling are in [3].

The main relation for the cutting tools force is:

$$F_z = C \cdot K \cdot t^x \cdot S^y \cdot (HB)^z . \qquad (1)$$

In this relation the coefficients and parameters which interfere are:

- C represents constant which takes values that depends on the nature of the material that is manufactured;
- $K = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \cdot K_6 \cdot K_7 \cdot K_8 \cdot K_9 \cdot K_{10} \cdot K_{11} \cdot K_{12}$

this is the global correctness coefficient, where:

$K_1 = K_v$ (*v* is the main speed of the cutting tool);

$K_2 = K_{ms}$ (*ms* represents the tools material);

$K_3 = K_{ma}$ (*ma* represents the cutting environment);

$K_4 = K_{mp}$ (*mp* represents the manufacture material);

$K_5 = K_\alpha$ (α is the angel of placement);

$K_6 = K_\gamma$ (γ is the angel of disengagement);

$K_7 = K_\lambda$ (λ is the bending angel of the active edge);

$K_8 = K_a$ (κ the attack angel);

$K_9 = K_r$ (*r* is the point of tooth cutting);

$K_{10} = K_\rho$ (*ρ* the sharpen ray);

$K_{11} = K_{ft}$ (*ft* represents the active edge of geometrical form);

$K_{12} = K_h$ (*h* it is the height of the usage surface).

In the mathematical point of view, it interests us the function variation:

$$f(t, s, HB) = C \cdot K \cdot t^x \cdot s^y \cdot (HB)^z,$$

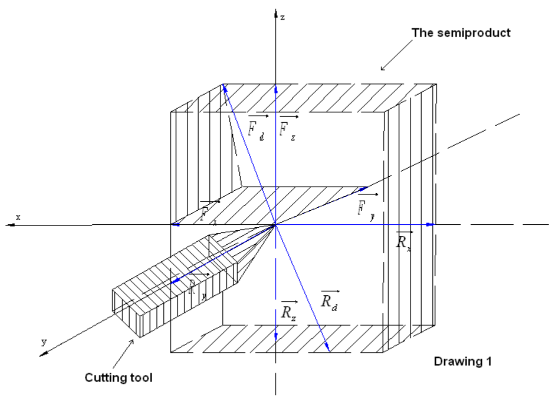
where: *C* and *K* are constants; *t* the de depth contribution of cutting; *s* the advance contribution of cutting; *(HB)* Brinnel hardness.

In this relation *x,y,z* are the levels of influence for the variables *t, s, HB*.

The constant values from this relation depend on the process of manufacture, the material nature of the cutting and hardness (C ≈ 0,56-70).

The cutting depth x ∈ [0.86;1.3] has 1 as the dominant value for cast iron and steel iron, cutting advance y ∈ [0.4;1] has the interval [0.7;0.8] as dominant and z ∈[0.35;0.75].

2. MATHEMATICAL MODEL



Drawing 1

$\vec{F_d}$ - the force of cutting.(F_d - the forces size);

$\vec{F_z}$ – the main component (F_z - his size);

$\vec{F_x}$ – the advance component(F_x -his size);

$\vec{F_y}$ – the radial component, normally at the manufactured area (F_y - his size);

$\vec{R_d}$ - cutting resistance;

$\vec{R_z}, \vec{R_x}, \vec{R_y}$ - the cutting resistance component.

In the relation the function is given *f = f(t, S, HB)*, x, y, and *z* are parameters that indicate the level of influence, and *t, S, HB* are in dependent variables.

We condition the variables t, S, HB by a relation which has the form: $t + S + HB = \alpha > 0$.

Therefore we must find de extremes of the function:

$$f = f(t, S, HB) = C \cdot K \cdot t^x \cdot S^y \cdot (HB)^z. \quad (2)$$

Relevantly: $t+S+HB = \alpha > 0$. (3)

The function ln: $(0,\infty) \to R$ is strictly ascending (the natural logarithm is in e base e≈2,718281828469) so the extremes of the function (2) conditioned by (3) will be the same as of the function F(t, S, HB)=lnf(t, S, HB), with relevantly (3).

Therefore we will look for the extremes of the function:

$$F(t, S, HB) = x\ln t + y\ln S + z\ln(HB) + \ln C + \ln K$$

subdued to the connection (3).

The multiplier of the Lagrange function is:

$$L(t, S, HB; \lambda) = F(t, S, HB) + \lambda(t+S+HB-\alpha) = x\ln t + y\ln S + z\ln(HB) + \ln C + \ln K + \lambda(t+S+HB-\alpha).$$

The stationary points of the function (5) are given by the system:

$$\begin{cases} L'_t = 0 \\ L'_S = 0 \\ L'_{HB} = 0 \\ L'_\lambda = 0 \end{cases} \Rightarrow \begin{cases} \frac{x}{t} + \lambda = 0 \\ \frac{y}{S} + \lambda = 0 \\ \frac{z}{HB} + \lambda = 0 \\ t + S + HB - \alpha = 0 \end{cases} \Rightarrow \begin{cases} t = -\frac{x}{\lambda} \\ S = -\frac{y}{\lambda} \\ HB = -\frac{z}{\lambda} \\ t + S + HB - \alpha = 0 \end{cases} \quad (4)$$

We obtain the multiply $\lambda_0 = \frac{x+y+z}{\alpha}$ and

$$t_0 = \frac{\alpha x}{x+y+z},\ S_0 = \frac{\alpha y}{x+y+z},\ HB_0 = \frac{\alpha z}{x+y+z}$$

Therefore

$M_0\left(\frac{\alpha x}{x+y+z}, \frac{\alpha y}{x+y+z}, \frac{\alpha z}{x+y+z}\right)$ is the stationary point for the function *F(t, S, HB)*, with the connection: $t+S+HB=\alpha \in [a,b] \subset (0,\infty)$.

The extremes conditioned by the connection of the function *L(t, S, HB; λ)*, are the same free extremes of the function *Φ(t, S, HB)=F(t, S, HB)+λ (t+ S+ HB-α)*.

The differential of the second order of the function Φ in the stationary point M_o is:

$$d^2\phi(M_0) = -\frac{(x+y+z)^2}{\alpha^2}\cdot\left[\frac{dt^2}{x}+\frac{dS^2}{y}+\frac{d(HB)^2}{z}\right]$$

From the connection (3) we obtain *d(HB)= -dt- dS* and then:

$$d^2\phi(M_0) = -\frac{(x+y+z)^2}{\alpha^2}\cdot$$

$$\left[\left(\sqrt{\frac{x+z}{xz}}dt+\sqrt{\frac{x}{z(x+z)}}dS\right)^2+\frac{x+y+z}{y(x+z)}dS^2\right]<0$$

.

We deduce that M_o is the maximum point conditioned of the function *f(t, S, HB)*.

The maximum value of the function *f(t, S, HB)* is:

$$f_{max} = f(t_0,S_0,HB_0) = C\cdot K\cdot\left(\frac{\alpha}{x+y+z}\right)^{x+y+z}\cdot x^x\cdot y^y\cdot z^z. \quad (5)$$

From: $t = \frac{\alpha x}{x+y+z}$, S= $\frac{\alpha y}{x+y+z}$, HB= $\frac{\alpha z}{x+y+z}$ we can determent the reciprocal relation, that is *x ,y, z* ,depending on *t, S, HB.*

The system

$$\begin{cases}(t-\alpha)\cdot x+ty+tz=0\\ Sx+(S-\alpha)\cdot y+Sz=0\\ (HB)\cdot x+(HB)\cdot y+(HB-\alpha)\cdot z=0\end{cases}$$

has the determinant:

$$\Delta = \begin{vmatrix} t-\alpha & t & t\\ S & S-\alpha & S\\ HB & HB & HB-\alpha\end{vmatrix} = (t+S+HB-\alpha)\begin{vmatrix} 1 & t & t\\ 1 & S-\alpha & S\\ 1 & HB & HB-\alpha\end{vmatrix} = 0,$$

because *t+S+HB=α.*

Therefore the system from above admits uneasily solution.

We obtain $\begin{cases} x=\frac{t}{HB}\mu\\ y=\frac{S}{HB}\mu\\ z=\mu\end{cases}$, cu $\mu\in R_+$, arbitrary.

We can determinate the extremes of the function *f(t,S,HB)=*$C\cdot K\cdot t^x\cdot S^y\cdot(HB)^z$, with the connection *t+S+HB=α* directly (without logarithm). For that we build the function:

L(t, S, HB; λ)= $C\cdot K\cdot t^x\cdot S^y\cdot(HB)^z$ $+\lambda\cdot(t+S+HB-\alpha)$

from:

$$\begin{cases} L_t' = C\cdot K\cdot x\cdot t^{x-1}\cdot S^y\cdot(HB)^z+\lambda=0\\ L_S' = C\cdot K\cdot y\cdot t^x\cdot S^{y-1}\cdot(HB)^z+\lambda=0\\ L_{HB}' = C\cdot K\cdot z\cdot t^x\cdot S^y\cdot(HB)^{z-1}+\lambda=0\\ L_\lambda' = t+S+HB-\alpha=0\end{cases} \Rightarrow \begin{cases} t=\frac{x}{y}S\\ HB=\frac{z}{y}S\\ t+S+HB=\alpha\end{cases}$$

. We find the same stationary point

$$M_0\left(t_0=\frac{\alpha x}{x+y+z}, S_0=\frac{\alpha y}{x+y+z}, HB_0=\frac{\alpha z}{x+y+z}\right),$$

adequate to the value:

$$\lambda_0 = -C\cdot K\cdot\left(\frac{\alpha}{x+y+z}\right)^{x+y+z-1}\cdot x^x\cdot y^y\cdot z^z$$ of

Lagrange's multiplication.

The second difference $d^2\phi(M_0)$ is a defined quadratic negative form, from were we deduce that *M* is a maximum conditioned point foot the function f(*t, S, HB*).

We obtain:

$$f_{max} = f_{(M_0)} = C\cdot K\cdot\left(\frac{\alpha}{x+y+z}\right)^{x+y+z}\cdot x^x\cdot y^y\cdot z^z$$

3. EXAMPLES

For: t_o = 1,5 mm, S_o = 0,15 mm/rot, *HB* ∈{150, 170, 200}, *K* ≈ 1 and *C* according to the Brinnel hardness, we obtain the maximum values for the main cutting force:

[1.] HB_o = 150 daN/mm^2, we obtain:

$F_{z\,max}$ = 110,079 daN;

[2.] HB_o = 170 daN/mm^2, we obtain:

$F_{z\,max}$ = 125, 132 daN;

[3.] $HB_o = 200$ daN/mm^2, we obtain:
$F_{z\,max} = 147,357$ daN;
[4.] $HB_o = 250$ daN/mm^2, we obtain:
$F_{z\,max} = 182,357$ daN.

For HB ≤ 170 dN/mm^2 we suggest the following law:

$$f(t,s,HB) = C \cdot K \cdot t^{\frac{6}{5}} \cdot s^{\frac{3}{4}} (HB)^{\frac{9}{20}}.$$

With K ≈ 1 and C ≈ 27,9 in the first two cases we obtain :
[1'] $HB_o = 150$ daN/mm^2, we obtain
$F_{z\,max} = 104,284$ daN;
[2'] $HB_o = 170$ daN/mm^2, we obtain
$F_{z\,max} = 110,326$ daN.

For HB > 170 dN/mm^2 we suggest the following law:

$$f(t,s,HB) = C \cdot K \cdot t^{\frac{3}{4}} \cdot s^{\frac{1}{3}} (HB)^{\frac{3}{4}}.$$

With K ≈ 1 and C ≈ 3,57 in the first two cases we obtain :
[3'] $HB_o = 200$ daN/mm^2, we obtain
$F_{z\,max} = 185,45$ daN;
[4'] $HB_o = 250$ daN/mm^2, we obtain
$F_{z\,max} = 219,234$ daN.

These values are closer to the values obtained through technic-experimental researches (the ones from [1], [2], [3] and [4]). In the second part of this paper we will return to some comparative tables and diagrams of the values of $F_{z\,max}$ depending on *HB*, for different sets of values given for *t* and *S*.

Bibliography

1. *Altintas, Y*. *Prediction cutting forces and tool breakage in milling from feed drive current measurements. Journal of Engineering for Industry, 113, nr.4, 1992.*

2. *Buzatu, C.* *ş.a. Sisteme flexibile de prelucrare prin aşchiere, Editura tehnică București, 1993.*

3. *Cozmâncă, M*., ***Panait, S***., ***Constantinescu,C***, *Bazele aşchierii, Editura "Gh.Asachi", IAŞI, 1995.*

4. *Croitoru, I., Segal,R., Cozmâncă,M*., *Model for predicting cutting forces. Buletinul V I.P.I, Tonul XLIV(XLVIII), Supliment I, Secţia V, 1998, pg. 189-193.*

5. *Gornyet,J.,F.*, *Physique et structures fractals, Masson, Paris, 1992.*

6. *Grasso,V.*, ***Noto la Diega, S***. ***Passannanti, A***. *Dynamic cutting coefficients in three-dimensional cutting. International Journal of Machine. Tools Design & Research, 20, nr.314,198.0.*

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