Tachyon-like Spectrum of Electromagnetic Modes in the Case of Extremely Low Frequencies in a System of Nanopores.

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***Abstract* — In nanotechnology, porous materials play an important role in achieving new properties. In this work we examine the electromagnetic behavior of a system of cylindrical nanopores, aranged in a two-dimmensional square lattice, at extremely low frequencies. Our geometry is considered a photonic crystal and then we use the plane wave expansion method to solve for the eigenvalues problem that arise after the Fourier transform of our initial representation, by doing so we actually find the dispersion relation that in turn gives us a tachyon-like spectrum to study .**

***Index Terms* — tachyon-like, ultra-low frequency, nanopore, photonic crystal, plane wave expansion.**

I. INTRODUCTION

In nanotechnology various types of nanopores are obtained. Our goal is to obtain the dispersion relation in a system of cylindrical nanopores of radius R, arranged in a periodic two-dimensional lattice with constant *a*.

II. THE EQUATIONS FOR A SYSTEM OF NANOPORES

Consider the propagation of electromagnetic waves in a system of simple cylindrical nanopores arranged in an infinite grid (we consider only the square). For simplicity, only the case where the component of the electric field $\vec{E}(\vec{r})$ along the axis of the nanopores is not equal to zero , where $\vec{r} = [X, Y, Z]$.

Description is based on the standard Maxwell's equations in vacuum [1]. From the Maxwell's equations, it follows that for a non-magnetic media, this situation, in the case of extremely low frequencies $\omega \to 0$ ($\lambda = 2\pi c/\omega >> a$) is fully described by the equations for the electric field $\vec{E}(\vec{r})$ $\left(\vec{H}(\vec{r}) \equiv 0\right)$.

$$\mathrm{div}\left(\varepsilon(\vec{r})\vec{E}(\vec{r})\right) = 0 \qquad (1)$$

$$\vec{E}(\vec{r}) = -\mathrm{grad}\left(\varphi(\vec{r})\right) \qquad (2)$$

where $\varepsilon(\vec{r})$ - the relative dielectric constant at point $\vec{r}$ **.**

The solution of the above equation is expressed as

$$F(\vec{r}) = F(\vec{\rho})\exp(ihz) \qquad (3)$$

Where h - wave attenuation along the tubes axis (z axis), $\vec{F}(\vec{\rho})$ - function describing the distribution along the plane (x,y) of any of the variables (ε, $\vec{E}$, φ), .

From equations (1,3) it follows that :

$$ih\varepsilon(\vec{\rho})E_z(\vec{\rho}) + \nabla_\perp\left(\varepsilon(\vec{\rho})E_\perp(\vec{\rho})\right) = 0 \qquad (4)$$

From equations (2,3) it follows that :

$$ih\vec{E}_\perp(\vec{\rho}) = \nabla_\perp\left(E_z(\vec{\rho})\right) \qquad (5)$$

Where $\nabla_\perp$ two-dimensional gradient operator, $\vec{E}_\perp = [E_x, E_y]$.

From equations (4,5) that the generalized equation for the eigenvalues h^2

$$h^2\varepsilon(\vec{\rho})E_z(\vec{\rho}) = \nabla_\perp\left(\varepsilon(\vec{\rho})\nabla_\perp E_z(\vec{\rho})\right) \qquad (6)$$

In fact, we have the operators equality in the coordinate representation. Note that equation (6) is completely equivalent mathematically to a two-dimensional problem of the conductivity of the dispersion medium at $\omega \to 0$.

It is only necessary in (6) to replace $E_z(\vec{\rho})$ by the value of the electric potential, and $\varepsilon(\vec{\rho})$ by the conductivity respectively.

The simplest solution of (6) is known :

$$h \equiv 0,\ E_z(\vec{\rho}) = \mathrm{const} \qquad (7)$$

Similar methods are used in describing the dispersion media to obtain their effective parameters [2]. We show that except for the simplest solutions (7), there are non-trivial solutions for $h \neq 0$ in (6).

In the following we consider only the case when the spatial distribution of the $\varepsilon(\vec{r})$ - relative permittivity for a single nanocylinder placed at the origin, is given by :

$$\varepsilon(\vec{r}) = \theta(R-\rho)(\varepsilon_1 - \varepsilon_2)|1-(\rho/R)^2|^\nu + \varepsilon_2 \qquad (8)$$

where θ (•)-step function, R-radius of the nanopore, ε2/ε1-permittivity outside/inside the nanocylinder (ε_1=1, ε_2=12). Parameter $\nu \geq 0$, describes the interface between nanocylinders and its surrounding space. The value $\nu = 0$ corresponds to a perfectly flat surface. Values $\nu > 0$ correspond to a more realistic model of the interface, including the transition layer thickness $L \sim R \cdot \nu$ (accounting irregularities, or other inhomogenities according to the conditions $L \ll R$, $2\pi/h$). A similar method is used in [3].

To find the eigenvalues and eigenfunctions of (6) we use the standard method of plane waves [4]. Solution of the equations for a system of nanotubes is found in the form of a Fourier expansion in two-dimensional plane waves

$$\vec{F}(\vec{\rho}) = \sum_{\vec{q}_n} \tilde{F}(\vec{k}+\vec{q}_n)\exp(i(\vec{k}+\vec{q}_n)\vec{\rho}) \qquad (9)$$

where

$$\tilde{F}(\vec{k}+\vec{q}_n) = \frac{1}{S}\iint_S d^2\vec{\rho}\, F(\vec{\rho})\exp(-i(\vec{k}+\vec{q}_n)\vec{\rho}) \qquad (10)$$

$\vec{q}_n$ - vector of the reciprocal two-dimensional lattice, $\vec{k}$ - vector of the two-dimensional motion, changes within the Brillouin zone, $S = a^2$ - area of the unit cell, *a*-lattice constant. So we actually turn to the momentum representation [5].

Substituting (9) into (6), we obtain an equation to determine the eigenvalues h^2 :

$$\mathrm{Det}(Z)=0 \qquad (11)$$

Where $Z = h^2\tilde{\varepsilon} - \tilde{A}$, $\tilde{\varepsilon}$ and $\tilde{A}$ are a matrices of the rank 300, so that :

$$(12) \qquad \varepsilon(\vec{k},\vec{k}') = \begin{cases} q = |\vec{k}-\vec{k}'| \equiv 0 \rightarrow [\varepsilon_2 + \pi(R/a)^2(1+\nu)(\varepsilon_1-\varepsilon_2)] \\ q = |\vec{k}-\vec{k}'| \neq 0 \rightarrow \pi\left(\frac{R}{a}\right)^2 2^{\nu+1}\Gamma(2+\nu)(\varepsilon_1-\varepsilon_2)J_{1+\nu}\frac{(qR)}{(qR)^{1+\nu}} \end{cases}$$

$$(13) \qquad \tilde{A}(\vec{k},\vec{k}') = \begin{cases} q=|\vec{k}-\vec{k}'|\equiv 0 \rightarrow -k^2[\varepsilon_2+\pi(R/a)^2(1+\nu)(\varepsilon_1-\varepsilon_2)] \\ q=|\vec{k}-\vec{k}'|\neq 0 \rightarrow -\pi\left(\frac{R}{a}\right)^2 \vec{k}\vec{k}' 2^{\nu+1}\Gamma(2+\nu)(\varepsilon_1-\varepsilon_2)J_{1+\nu}\frac{(qR)}{(qR)^{1+\nu}} \end{cases}$$

J (•), Γ (•) - specific Bessel and Gamma functions [6]. Further, for simplicity, we restrict ourselves to the case of motion along the pores ($\vec{k} = 0$). The Z matrix in the equation (11) is expanded in a power series in R up to the five term to attain numerical results with 300-rank matrixs. The graphs of the dependence of the wave vector $h=h(R)\neq 0$ on the radius R of photonic crystals of nanopores are obtained by using the MATLAB's polyeig function [7]

III. RESULTS

The solution of the obtained eigenvalue problem leads to the following dependence for nonzero wave vectors ($h \neq 0$) of the pore radius R (where $\nu = 0.1$).

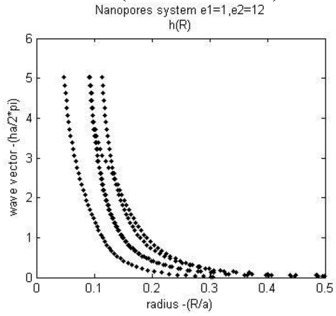
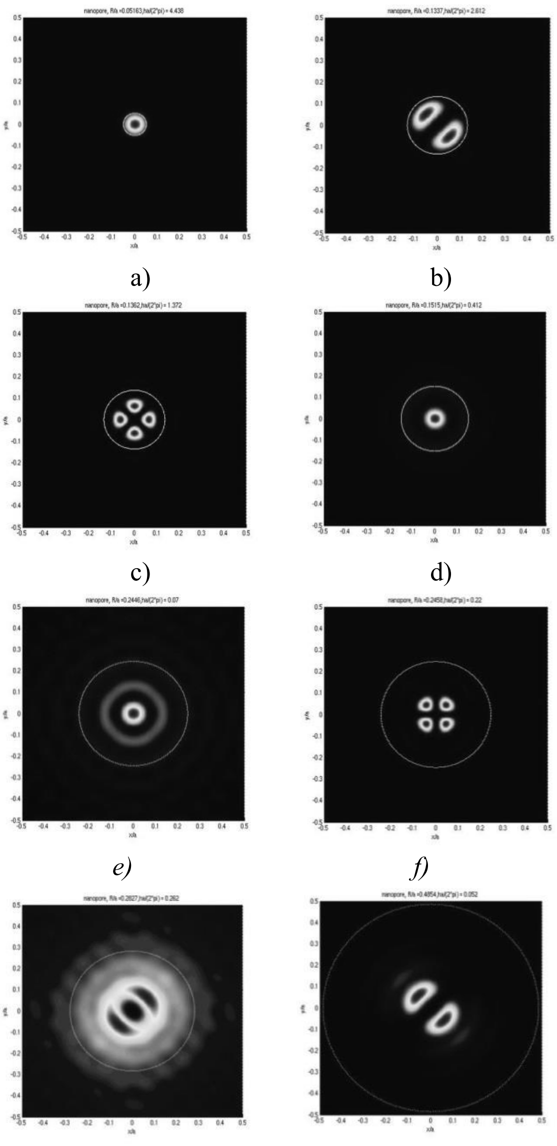


Fig. 1: The dependence for nonzero wave vectors ($h \neq 0$) on the pore radius R.

Individual modes have the following distribution of $|E_z(\vec{\rho})|^2$ in the section plane pores with different R and h :



a) b) c) d) *e)* *f)* g) h)

Fig. 3 The distribution of the intensity $|E_z(\vec{\rho})|^2$ with different values of the parameters R and h :

a) R/a=0.051 , ha/(2π)=4.438;
b) R/a=0.133 , ha/(2π)=2.612;
c) R/a=0.136 , ha/(2π)=1.372;
d) R/a=0.151 , ha/(2π)=0.412;
e) R/a=0.244 , ha/(2π)=0.070;
f) R/a=0.245 , ha/(2π)=0.220;
g) R/a=0.282 , ha/(2π)=0.262;
h) R/a=0.485 , ha/(2π)=0.052;

In Fig. 3 we clearly see that the two parameters R and h are in a confrontation with each other, i.e. their relation, causes the formation of specific symmetries of the modes, particularly, by shrinking the radius R the modes tend to become somewhat localized and at relatively small values collapse into one single domain, whereas by varying h it seems there is a tendency to cause some perturbations in the modes while decreasing h (with R=const) .

Since we consider the case of extremely low frequency, wavelength similar fashion $\lambda_h << \lambda$ ($\lambda = 2\pi c / \omega$ - vacuum wavelength). Therefore, these modes can be called ultrashort[8]. Frequency dispersion for these modes is shown on the figure below :

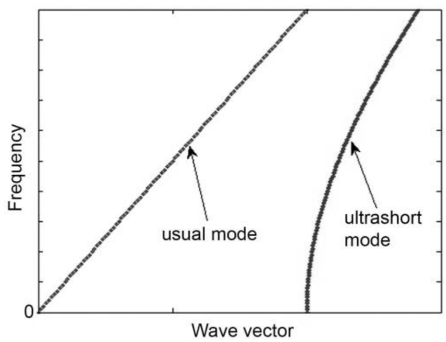


Fig. 3: A schematic view of a Tachyon-like dispersion relation.

It has a tachyon-like character, i.e. has a group velocity much greater than the speed of light at $\omega \rightarrow 0$. This fact does not contradict the principles of the theory of relativity, because in this mode has no magnetic component, and it does not transfer energy [9].

IV. CONCLUSION

In this paper the possibility of tachyon-like ultra-low-frequency modes in a system of nanopores in the case of extremely low frequency is exposed by traditional methods used in the theory of photonic crystals.

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