Controllers to the Structures of Model Objects in Parallel Conection

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***Abstract* —In this paper is proposed the iterative algorithm of tuning the typical controllers PI and PID to the structures of model objects in the parallel connection that presents the types of model object with anticipation and delay second order and third order. The proposed algorithms of tuning the controllers use the maximal stability degree method with iteration. In the result of this study is proposed the algorithm of tuning controllers and the procedure of determination the performances of control system in the dependency of the maximal stability degree value of the designed system.**

***Index Terms* —the structures of model objects connected in parallel; the model objects with anticipation-delay; controller; the tuning of controller; the maximal stability degree method with iteration.**

I. INTRODUCTION

At the automation the diverse industrial processes the mathematical models attached to this processes are presented as structures of complex models. For the approximation the evolution of processes the most uses mathematical models are characterized with inertia of process [1, 2].

In this paper is analyzed the model objects of the industrial processes which structures are presented in the parallel connection, composed from two and three elements with inertia first order and is analyzed the two cases:

- the case when the elements are not identical;
- the case when the elements are identical.

The transfer function of the model object in the parallel connection with two elements with inertia first order is presented in the following form

$$H(s) = \frac{k_1}{T_1 s+1} + \frac{k_2}{T_2 s+1} = \frac{b_0 s+b_1}{a_0 s^2+a_1 s+a_2}, \qquad (1)$$

where k_1, k_2 are the transfer coefficients, T_1, T_2 – the time constants $b_0 = k_1 T_2 + k_2 T_1$, $b_1 = k_1 + k_2$, $a_0 = T_1 T_2$, $a_1 = T_1 + T_2$, $a_2 = 1$.

The transfer function of the model object in the parallel connection with three elements with inertia first order is presented in the following form

$$H(s) = \frac{k_1}{T_1 s+1} + \frac{k_2}{T_2 s+1} + \frac{k_3}{T_3 s+1} = \frac{b_0 s^2+b_1 s+b_2}{a_0 s^3+a_1 s^2+a_2 s+a_3}, \qquad (2)$$

where k_1, k_2, k_3 are the transfer coefficients,

T_1, T_2, T_3 – the time constants,

$b_0 = k_1 T_2 T_3 + k_2 T_1 T_3 + k_3 T_1 T_2$,

$b_1 = k_1(T_2 + T_3) + k_2(T_1 + T_3) + k_3(T_1 + T_2)$,

$b_2 = k_1 + k_2 + k_3$,

$a_0 = T_1 T_2 T_3$, $a_1 = T_1 T_2 + T_1 T_3 + T_2 T_3$,

$a_2 = T_1 + T_2 + T_3$, $a_3 = 1$.

In the case when in the relations (1) and (2) the elements are identical, these relations will have the following form

$$H(s) = \frac{k}{Ts+1} + \frac{k}{Ts+1} = \frac{2k(Ts+1)}{(Ts+1)^2} = \frac{b_0 s+b_1}{a_0 s^2+a_1 s+a_2}, \qquad (3)$$

where $b_0 = 2kT$, $b_1 = 2k$, $a_0 = T^2$, $a_1 = 2T$, $a_2 = 1$.

$$H(s) = \frac{k}{Ts+1} + \frac{k}{Ts+1} + \frac{k}{Ts+1} = \frac{b_0 s^2+b_1 s+b_2}{a_0 s^3+a_1 s^2+a_2 s+a_3} = \qquad (4)$$
$$= \frac{3k(Ts+1)^2}{(Ts+1)^3},$$

where $b_0 = 3kT^2$, $b_1 = 6kT$, $b_2 = 3k$, $a_0 = T^3$,

$a_1 = 3T^2$, $a_2 = 3T$, $a_3 = 1$.

The zeros and poles of expression (3) and (4) are equal (with -1/*T*), from this is result that the one zero-pole and two zero-pole are compensated, and the dynamic of this model objects are presented as model objects with inertia first order with following transfer function.

$$H(s) = \frac{2k}{Ts+1}, \qquad (5)$$

$$H(s) = \frac{3k}{Ts+1}. \qquad (6)$$

The expressions (5) and (6) represent the transfer functions of the equivalent model objects (3) - (4) and there is the model object with inertia first order. At the denominator of these expressions, the transfer coefficient represents production between transfer coefficient *k* of the identical element and the number that indicates the number of elements in the parallel connection.

The model objects (1)-(4) are presented the model objects with anticipation-delay. To this model objects are proposed to tune the PI and PID controllers using the maximal stability degree method with iteration [3-6]. It is analyzing the examples of tuning the PI and PID controllers to the model objects (1)-(4). It was varying the parameters of model object and was verifying the robustness of synthesized control system.

To the model object with known parameters b_0, b_1, b_2, a_0, a_1, a_2, a_3 is proposed to tune the typical controllers PI and PID use the maximal stability degree

(MSD) method [4-6] and is proposed to make a variation of object parameters from the nominal values, keeping the tuning values of PI and PID controller to analyze the dynamics and the robustness of automatic control system.

II. THE TUNING ALGORITHM OF CONTROLLERS

Assume that the control system is formed of object with transfer function $H_{PF}(s)$ and transfer function of controller $H_R(s)$ with typical control laws PI, PID (Fig. 1).

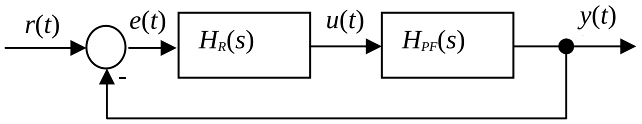


Fig. 1. The structure scheme of control system.

It will be tune the typical algorithms of control PI and PID to the model object with known parameters presented by the relation (2), using the MSD method [4-6].

For the tuning the PI controller to the model object non - identical elements, using the MSD method was applying the algebraic expressions, which are presented the analytical expressions in the form

$$k_p = \frac{c_0J^5 - c_1J^4 + c_2J^3 - c_3J^2 + c_4J - c_5}{d_0J^4 - d_1J^3 + d_2J^2 - d_3J + d_4}, \qquad (7)$$

where

$c_0 = 2a_0b_0, c_1 = 3a_0b_1 + a_1b_0, c_2 = 4a_0b_2 + 2a_1b_1,$

$c_3 = 3a_1b_2 + a_2b_1 - a_3b_0, \; c_4 = 2a_2b_2, c_5 = a_3b_2,$

$d_0 = b_0^2, d_1 = 2b_0b_1, d_2 = 2b_0b_2 + b_1^2, d_3 = 2b_1b_2, d_4 = b_2^2;$

$$k_i = \frac{-a_0J^4 + a_1J^3 - a_2J}{b_0J^2 - b_1J + b}J. \qquad (8)$$

For the tuning the PID controller to the model object with non-identical elements, using the MSD method was obtained the algebraic expressions for the tuning parameters of the controller which are presented the analytical expressions in the form

$$k_d = \frac{-c_0J^8 + c_1J^7 - c_2J^6 + c_3J^5 - c_4J^4 + c_5J^3 - c_6J^2 + c_7J - c_8}{(d_0J^4 - d_1J^3 + d_2J^2 - d_3J + d_4)^2}, \qquad (9)$$

where $c_0 = 2a_0b_0^3, \; c_1 = 8a_0b_0^2b_1,$

$c_2 = 8a_0b_0^2b_2 + 12a_0b_0b_1^2,$

$c_3 = 28a_0b_0b_1b_2 + 6a_0b_1^3 - 2a_1b_0^2b_2 + 2a_1b_0b_1^2 -$

$-2a_2b_0^2b_1 + 2a_3b_0^3,$

$c_4 = 18a_0b_0b_2^2 + 22a_0b_1^2b_2 + 4a_1b_0b_1b_2 + 2a_1b_1^3 -$

$-2a_2b_0b_1^2 - 6a_2b_0^2b_2 + 2a_3b_0^2b_1,$

$c_5 = 28a_0b_1b_2^2 + 4a_1b_0b_2^2 + 3a_1b_1^2b_2 - 8c_2b_0b_1b_2 - 4a_3b_0^2b_2,$

$c_6 = 12a_0b_2^3 + 12a_1b_1b_2^2 - 4a_2b_0b_2^2 - 8a_3b_0b_1b_2,$

$c_7 = 6a_1b_2^3 + 2a_2b_1b_2^2 - 6a_3b_0b_2^2 - 2a_3b_1^2b_2,$

$c_8 = 2a_2b_2^3 - 2a_3b_1b_2^2,$

$$k_p = \frac{c_0J^5 - c_1J^4 + c_2J^3 - c_3J^2 + c_4J - c_5}{d_0J^4 - d_1J^3 + d_2J^2 - d_3J + d_4} + 2k_dJ, \qquad (10)$$

where coefficients from expression (8) have the form of coefficients from expression (5):

$c_0 = 2a_0b_0, c_1 = 3a_0b_1 + a_1b_0, c_2 = 4a_0b_2 + 2a_1b_1,$

$c_3 = 3a_1b_2 + a_2b_1 - a_3b_0, \; c_4 = 2a_2b_2, c_5 = a_3b_2,$

$d_0 = b_0^2, d_1 = 2b_0b_1, d_2 = 2b_0b_2 + b_1^2, d_3 = 2b_1b_2, d_4 = b_2^2;$

$$k_i = \frac{-a_0J^4 + a_1J^3 - a_2J^2 + a_3J}{b_0J^2 - b_1J + b_2} - k_dJ^2 + k_pJ. \qquad (11)$$

For the case of object with identical elements the coefficients from expression (5) for PI controller have the following form

$$c_0 = 6kT^5, c_1 = 27kT^4, c_2 = 48kT^3, c_3 = 42kT^2,$$
$$c_4 = 18kT, c_5 = 3k; \; d_0 = 9k^2T^4, d_1 = 36k^2T^3, \qquad (12)$$
$$d_2 = 54k^2T^2, d_3 = 36k^2T, d_4 = 9k^2.$$

For the case of object with identical elements the coefficients from expression (5) for PID controller have the following form

$$c_0 = 54k^3T^9, c_1 = 432k^3T^8, c_2 = 1512k^3T^7,$$
$$c_3 = 3024k^3T^6, c_4 = 3780k^3T^5, c_5 = 3024k^3T^4,$$
$$c_6 = 1512k^3T^3, c_7 = 432k^3T^2, c_8 = 54k^3T, \qquad (13)$$
$$d_0 = 9k^2T^4, d_1 = 36k^2T^3, d_2 = 54k^2T^2,$$
$$d_3 = 36k^2T, \; d_4 = 9k^2.$$

From expressions (7) and (8) were determined the optimal values of parameters k_p and k_i of the PI controller.

From expressions (9), (10) and (11) were determined the optimal values of parameters k_p, k_i and k_d of PID controller. Tuning the PID controller to the model object with identical elements (4) is not applicable, this is explaining from the reason that the numerator coefficients from expression (9) are binominal (13) and the parameters are compensated reciprocally.

It will be tuned the typical control algorithms PI and PID to the equivalent model object with known parameters that is presented by the relation (6), using the MSD method [4, 5].

For the tuning the PI controller by the MSD method are obtained the following algebraic expressions that represent the analytical expressions

$$k_p = \frac{1}{nk(2TJ - 1)}, \qquad (14)$$

$$k_i = \frac{1}{nk}(-TJ^2 + J) + k_pJ = \frac{TJ^2}{nk}, \qquad (15)$$

where the n coefficient presents the number of identical elements in the models (5) or (6).

For the tuning the PID controller by the MSD method are obtained the following algebraic expressions that represent the analytical expressions

$$k_d = \frac{-2TJ + 1 + nkk_p}{2nkJ} = \frac{-2TJ + 1 + (nk)^2}{2nkJ}, \qquad (16)$$

$$k_i = \frac{1}{nk}(-TJ^2 + J) - k_dJ^2 + k_pJ = \frac{J}{2nk}(1 + (nk)^2). \qquad (17)$$

In the expressions (16), (17) was admitted that tuning parameter of the proportional part is

$$k_p = nk. \qquad (18)$$

From expressions (7)-(8) and (14)-(15) are determinate the optimal values k_p and k_i of the PI controller for the model object (2) and respectively for the equivalent model object (6).

From expressions (9)-(11) and (16)-(18) are determinate the optimal values of parameters k_p, k_i and k_d of the PID controller for the model object (2) and respectively for the equivalent model object (6).

The tuning parameters of PI and PID controller - k_p , k_i and k_d are the functions of known parametrs of control object and of the stability degree J that is unknown of the control system: $k_p=f(J)$, $k_i=f(J)$ and $k_d=f(J)$ (view relations (7)-(11) and (14)-(18)). Based on these relations in the case of known object's parameters and in the case of variation stability degree $J\geq 0$ in the respectively limits it was made the calculations and were obtained the dependences $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for PI and PID controllers.

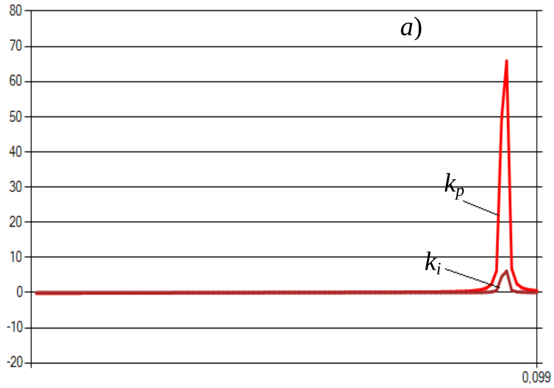
To obtain the setted performance of control system with respectivelly controller for the obtained curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ it is chosen the value sets of k_p, k_i şi k_d - J parameters for the different values of the J, and it is done the computer simulation of the control system and by the transient process is is determinated performances of control system. This procedure will repeat until the performance of system will be satisfied.

III. APPLICATION AND COMPUTER SIMULATION

To verify the proposed method of tuning the PI and PID controllers to the model object (2) it was assumed that the control object has the arbitrary values of the object's parameters:

1. The case for the non identical elements:
$k_1=2,\ k_2=3, k_3=0{,}2, T_1=10\ s, T_2=12\ s, T_3=5s$;
$b_0=294, b_1=83.4,\ b_2=5.2,\ a_0=600,$
$a_1=230, a_2=27, a_3=1,$
2. The case for the identical elements:
$k_1=\ k_2=k_3=0{,}5, T_1=T_2=T_3=10\ s,$
$b_0=150, b_1=30, b_2=1{,}5;\ a_0=1000,$
$a_1=300,\ a_2=30, a_3=\ 1.$
3. The case of equivalent model object:
$k=0{,}5$, $T=10$, $n=3$.

It was done the respectively calculation for the PI controller and were obtained $k_p=f(J)$, $k_i=f(J)$ dependencies for the following cases: Fig. 2, *a* – tuning the PI controller to the non-identical model object; Fig. 2, *b* – tuning the PI controller to the identical model object. In the Fig. 3 are presented the dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for the case of tune the PID controller to the non-identical model object.



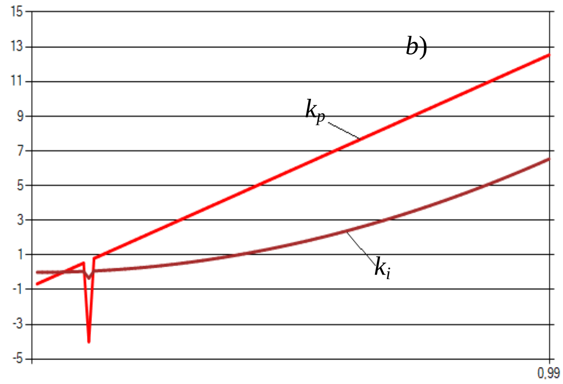


Fig. 2. The dependences of the PI controllers' parameters of the stability degree value.

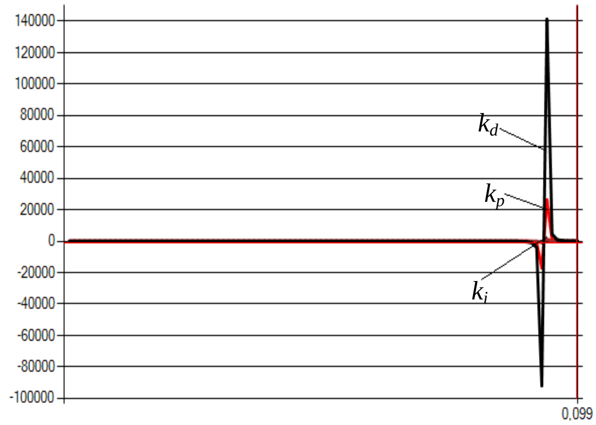


Fig. 3. The dependence of PID controllers' parameters of the stability degree value.

It was done the respectively calculation for the PI and PID controllers and in the Fig. 4 *a*, *b* are presented the dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$, for the case of identical equivalent model object: Fig. 4, *a* – tuning the PI controller, Fig. 4, *b* – tuning the PID controller.

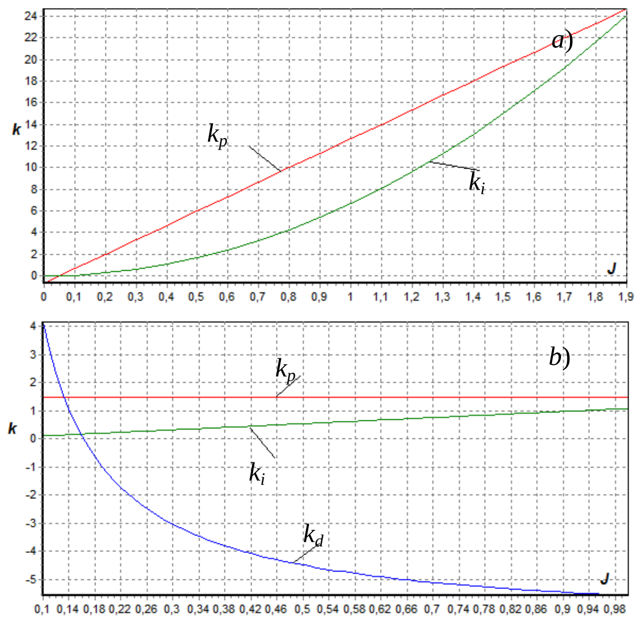


Fig. 4. The dependence of PID controllers' parameters of the stability degree value.

From Fig. 2, *a, b* were chosen the different sets of tuning values for PI controller that are presented in the Table I (the model object with non-identical elements) and Table II (case of model object with identical elements). It was done the computer simulation in MATLAB, the obtained results in case of tune the PI controller are presented in the Fig. 5, *a*, *b*: in Fig. 5, *a* are presented the transition processes of control system with non-identical elements; in the fig. 5, *b* are presented the transition processes of control system with identical elements. The curves 1-5 were obtained for the case of tune PI controller by the MSD method and curve 6 was obtained for the case of tune PI controller use the NCD Outport Block from MATLAB, curve 7 represents the transient process of the system with PI controller tuning to the equivalent model object (parameters presented in the row 6 from Table II).

TABLE I. THE VALUES OF THE PI CONTROLLER'S PARAMETERS IN CASE OF NON IDENTICAL MODEL OBJECT

No. item.	***J***	k_p	k_i
1	0,09	2,40	0,21
2	0,18	3,31	0,53
3	0,69	2,57	0,98
4	0,81	3,10	1,34
5	0,9	3,47	1,66
6		9.45	6.066

TABLE II. THE VALUES OF THE PI CONTROLLER'S PARAMETERS IN CASE OF IDENTICAL MODEL OBJECT

No. item.	***J***	k_p	k_i
1	0,65	8,0	2,82
2	0,72	8,93	3,46
3	1,05	13,33	7,35
4	1,47	18,93	14,41
5	1,89	24,53	23,81
6		44.30	25.58
7	1	12,67	6,67

From Fig. 3 were chosen the different sets of tuning values for PID controller that are presented in the Table III (the model object with non-identical elements). It was done the computer simulation in MATLAB and the obtained transient processes are presented in the Fig. 8. For the case of tuning the PID controller to the model object with identical elements the tuning parameters can not be calculated by the proposed method. This is explaining from the reason that the denominator coefficients from expression (7) are binominal and the parameters are compensated reciprocally. In Fig. 8 curve 5 represents the transient process of the system with PID controller tuning to the equivalent model object (parameters presented in the row 5 from Table III).

TABLE III. THE VALUES OF THE PID CONTROLLER'S PARAMETERS IN CASE OF NON IDENTICAL MODEL OBJECT

No. item.	***J***	k_p	k_i	k_i
1	0.095	181,6	8,8	942
2	0.097	31,3	1,56	156,5
3	0.098	17,21	0.88	84,2
4	0.099	10,45	0.55	50
5	0,16	1,5	0,173	0,104

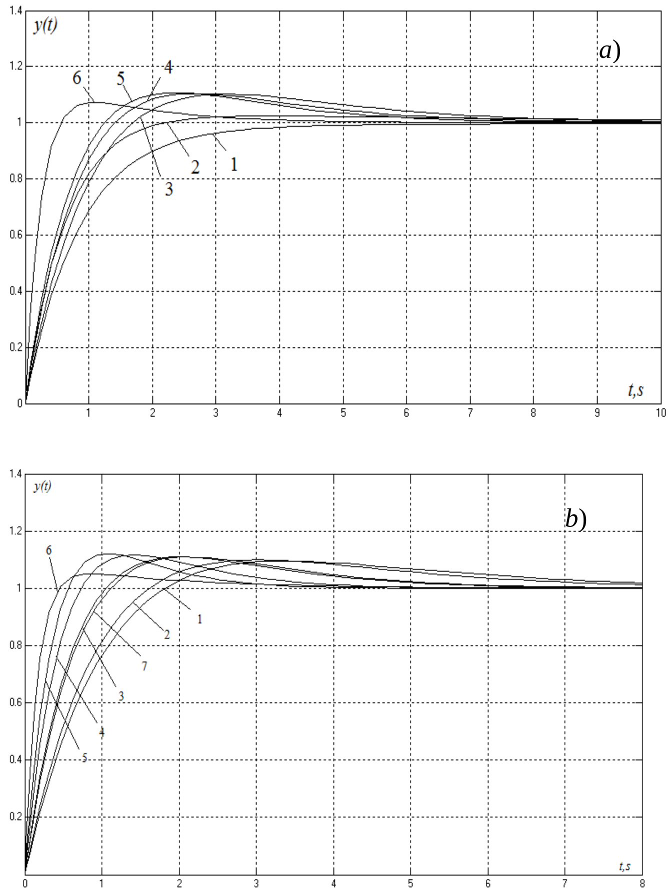


Fig. 5. The transient processes of control systems.

In the Table IV are presented the performances of control system with PI controller tuned to the non-identical model object and in the Table V are presented the performances of control system with PI controller tuned to the identical model object.

TABLE IV. PERFORMANCE OF CONTROL SYSTEM WITH PI CONTROLLER WITH NON-IDENTICAL ELEMENTS

Perform. of CS	**Nr. of curves**	ε, %	t_c, s	σ, %	t_r, s	λ
MSD Method	1	5	2.8		2.8	
	2	5	1.65	2.3	1.65	
	3	5	1.45	10.2	5.4	1
	4	5	1.25	10.4	4.5	1
	5	5	1.1	10.7	4.35	1
MATLAB Optimiz.	6	5	0.48	7.2	1.85	1

TABLE V. PERFORMANCE OF CONTROL SYSTEM WITH PI CONTROLLER WITH IDENTICAL ELEMENTS

Perform. of CS	**Nr. of curves**	ε, %	t_c, s	σ, %	t_r, s	λ

MSD Method	1	5	1.7		5	1
	2	5	1.8	1.2	4.9	1
	3	5	0.92	11	4.1	1
	4	5	0.65	11.7	3.5	1
	5	5	0.49	12.1	2.4	1
MATLAB Optimizat ion	6	5	0.36	5	2.2	1
MSD Method	7	5	0.91	1.17	4.05	1

Analyzing the obtained performances it can be observed that for the control system with PI controller tuned to the model object with three non identical elements in parallel connection the optimal transient process was obtained for the case of curves 2 and 6 from Fig. 5, *a* (the control time is t_r=1.65 s. and t_r=1.85 s.). For the case of tuning the PI controller to the model object with identical elements the optimal transient process was obtained for the case of curves 5 and 6, Fig. 5, *b* (the control time t_r=2.4 and t_r=2.2 s).

In the Fig. 6 is presented the comparison of transient processes and in the Fig. 7 is presented the distribution of the characteristic equation's poles, obtained for the case of tune the PI controller, in this figures are used the following notation: 1 - case of tune controller by the MSD method to the non identical model object; 2 - case of tune controller use NCD Outport Block from MATLAB to the non identical model object; 3 - case of tune controller by the MSD method to the identical model object; 4 - case of tune controller use NCD Outport Block from MATLAB to the identical model object.

Analyzing the obtained results it can be observed that the best result was obtain for the case of tune the PI controller to the non-identical elements and the system in this case has the reserve of stability higher than the system with identical elements.

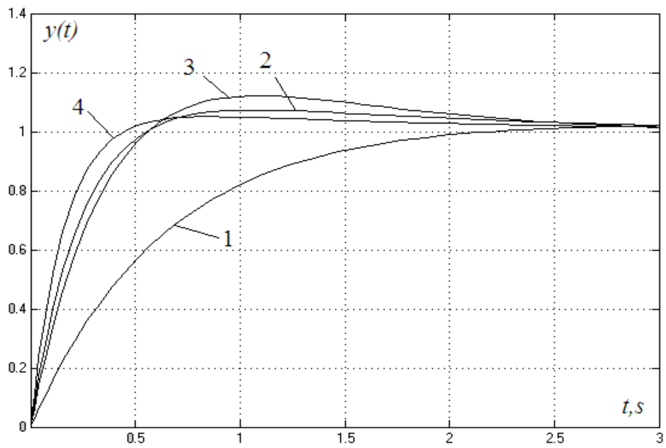


Fig. 6. The transient processes of control systems.

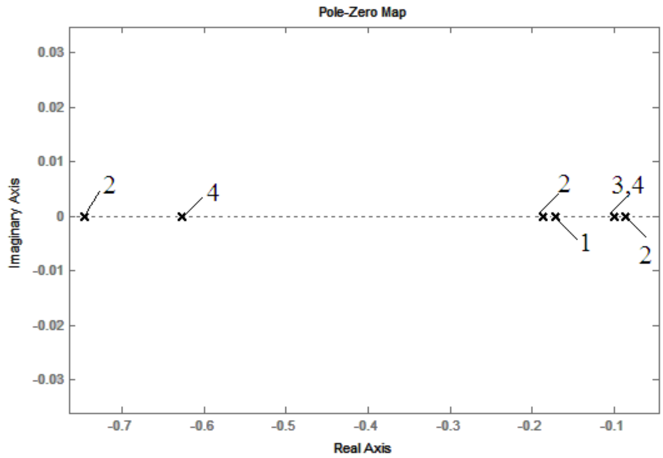


Fig. 7. The distribution of the characteristic equation's poles.

In the Fig. 8 are presented the transient processes obtained in the case of tune the PID controller to the non identical elements and the performances of control system are presented in the Table VI.

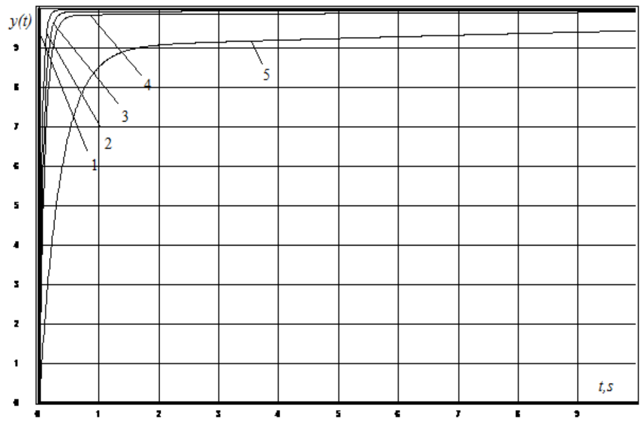


Fig. 8. The transient processes of control systems.

TABLE VI. PERFORMANCE OF CONTROL SYSTEM

Perfor m. of CS	**Nr. of curves**	ε, %	t_c, s	σ,%	t_r, s	
MSD Method	1	5	0.01	-	0.01	-
	2	5	0,13	-	0,13	-
	3	5	0,21	-	0,21	-
	4	5	0,31	-	0,31	-
	5	5	2	-	2	-

Analyzing the transient processes of the system with PID controller tuned to the model object with three non-identical elements in parallel connection the optimal transient process is presented by the curve 1 from Fig. 8.

In the case of control system with PI controller at the variation the J=0,1…1,0 … the performances are varying:

- Control system with model object (2) with non-identical elements: t_c=2,65…1,07, σ=0…10,75, t_r=2,65…4,34.
- Control system with model object (2) with identical elements: t_c=1,55…0,48, σ=9,62…12,2 %, t_r=5,8…2,12.
- Control system with equivalent model object (6): t_c=29,6…0,7, σ=0…13,75 %, t_r =29,6…3,6.

In the case of control system with PID controller at the variation the J=0,1…1,0 … the performances are varying:

- Control system with model object (2) with non-identical elements: t_c=0,01…0,44, σ=0, t_r=0,01…0,44.

- To the control system with model object (2) with identical elements the tuning of the PID controller is not applicable.

- Control system with equivalent model object (6): t_c=25,3…10,8, σ=0, t_r =25,3…10,8.

At the variation the parameters of the model object, the control system with controller tuned by the proposed method is robust.

IV. CONCLUSION

Analyzing the proposed results it can be done the following conclusions:

- It is proposed the graphic - analytical method of tuning the PI and PID controllers to the structures of models object with inertia first order in parallel connection that allows to obtain the desired performances of designed control system.
- For the control system with PI controller tuned to the model object with three non-identical elements in parallel connection the optimal transient process it was obtained for the case of curves 2, 6 (Fig. 5, *a*).
- For the case of tuning the PI controller to the model object with identical elements the optimal transient process was obtained for the case of curves 5 and 6, Fig. 5, *b*.
- For the case of control system with PID controller tuned to the model object with three non-identical elements in parallel connection the optimal transient process was obtained for the case of curve 1, Fig. 8, and for the case of tune the PID controller to the identical elements the method can not be applied.
- Analyzing the obtained transient processes for the cases of tune the PI and PID controllers, can be concluded that the highest performances were obtained for the case of tune the PID controller to the non-identical elements (the system has the control time t_r=0.01 s.).
- Control system with equivalent model object (6) and PI controller has the lower performances (the transient process is slow) than the control system with model object with non-identical and identical elements with PI controller (the transient process is faster).
- Control system with equivalent model object (6) and PID controller has the lower performances (the transient process is slow) than the control system with model object with non-identical elements with PID controller (the transient process is faster).

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