Mathematical modeling of seismic impact on buried projectile

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***Abstract* — The paper describes mathematical model and numerical results of the study of seismic impact on the buried in the ground projectile with fuse. Projectiles with fuse belong to potentially dangerous objects. The computer estimation of operational condition of potentially dangerous objects is actual problem for Moldova, as well as for other regions. Their damage or destruction in the event of seismic impact (or other force majeure) may lead to environmental disasters. Full-scale physical tests in the industry is not always possible or expensive, therefore increases the relevance of mathematical modeling.**

***Index Terms* — elastoplastic media, numerical methods, saturated ground, seismic effects modeling.**

I. INTRODUCTION

Today the new knowledge retrieval in the area of technological risks assessment and the creation of new technologies, services and products have directly related to the application of mathematical modeling and use of multiprocessor computer systems.

The creation of high-performance computing environment and effective software applications are important scientific and practical results that allows high-precision simulation and visualization of complex physical processes and objects by mathematical modeling without full-scale experiments [1].

It fully applies to a wide range of problems of solid mechanics with the influence of various physical effects and properties characteristics of construction materials [2-4]. Modeling of such problems is characterized by high demands to computing resources.

II. PROBLEM FORMULATION AND MATHEMATICAL MODEL

The two-dimensional model of elastic-plastic medium has been chosen to describe the behavior of the structural material and explosive substance (ES). The given model belongs to the class of models with internal state parameters and bases on thermodynamic principles of continuum mechanics.

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} = \frac{\partial(\rho \upsilon)}{\partial r} - \frac{\partial(\rho \upsilon)}{\partial z} - \frac{\rho u}{r},\\
&\frac{\partial(\rho u)}{\partial t} = -\frac{\partial P}{\partial r} + \frac{\partial s_{rr}}{\partial r} + \frac{s_{rr} - s_{\theta\theta}}{r},\\
&\frac{\partial(\rho \upsilon)}{\partial t} = -\frac{\partial P}{\partial z} + \frac{\partial s_{rr}}{\partial z} + \frac{\partial s_{zr}}{\partial r} + \frac{s_{rz}}{r},\\
&\frac{\partial(\rho E)}{\partial t} = -\frac{P}{\rho}\frac{\partial \rho}{\partial t} + s_{rr}\frac{\partial u}{\partial r} + s_{\theta\theta}\frac{u}{r} + s_{rz}\left(\frac{\partial u}{\partial z} + \frac{\partial \upsilon}{\partial r}\right),
\end{aligned} \tag{1}$$

here ρ is density, $\{u, \upsilon\} = \bar{\omega}$ are velocity vector components, P is hydrostatic pressure, $\sigma_{i,j} = -P\delta_{i,j} + s_{i,j}$, where $\sigma_{i,j}$ is stress tensor, E is internal energy per original volume. The stress tensor deviator components associated with the strain rate tensor $\varepsilon_{i,j}$ as follows:

$$\begin{aligned}
&\dot{s}^{\nabla}_{zz} = 2\mu\left(\dot{\varepsilon}_{zz} + \frac{1}{3\rho}\frac{\partial \rho}{\partial t}\right),\\
&\dot{s}^{\nabla}_{rr} = 2\mu\left(\dot{\varepsilon}_{rr} + \frac{1}{3\rho}\frac{\partial \rho}{\partial t}\right),\\
&\dot{s}^{\nabla}_{\theta\theta} = 2\mu\left(\dot{\varepsilon}_{\theta\theta} + \frac{1}{3\rho}\frac{\partial \rho}{\partial t}\right), \dot{s}^{\nabla}_{zz} = \mu(\dot{\varepsilon}_{rz})
\end{aligned} \tag{2}$$

here ∇ is Jaumann derivative:

$$\begin{aligned}
&\dot{s}^{\nabla}_{ij} = s_{ij} - s_{ik}\omega_{jk} - s_{jk}\omega_{ik},\\
&\omega_{ij} = \frac{1}{2}\left(\frac{\partial \upsilon_i}{\partial x_j} - \frac{\partial \upsilon_j}{\partial x_i}\right)
\end{aligned} \tag{3}$$

υ_i are velocity components, x_j are space coordinates. Velocity strains are defined as:

$$\begin{aligned}
&\dot{\varepsilon}_{zz} = \frac{\partial \upsilon}{\partial z}, \dot{\varepsilon}_{rr} = \frac{\partial u}{\partial r},\\
&\dot{\varepsilon}_{\theta\theta} = \frac{\upsilon}{r},\\
&\dot{\varepsilon}_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial r} - \frac{\partial \upsilon}{\partial z}\right)
\end{aligned} \tag{4}$$

Prandtl-Reuss equations of plasticity theory are

$$\begin{aligned}
&\frac{\partial s_r}{\partial t} + v\frac{\partial s_r}{\partial r} + u\frac{\partial s_r}{\partial z} + \lambda s_r = 2G\left(\frac{\partial v}{\partial r} - \frac{1}{3\rho}\frac{\partial \rho}{\partial t}\right),\\
&\frac{\partial s_\varphi}{\partial t} + v\frac{\partial s_\varphi}{\partial r} + u\frac{\partial s_r}{\partial z} + \lambda s_\varphi = 2G\left(\frac{v}{r} - \frac{1}{3\rho}\frac{\partial \rho}{\partial t}\right),\\
&\frac{\partial s_z}{\partial t} + v\frac{\partial s_z}{\partial r} + u\frac{\partial s_z}{\partial z} + \lambda s_z = 2G\left(\frac{\partial u}{\partial z} - \frac{1}{3\rho}\frac{\partial \rho}{\partial t}\right),\\
&\frac{\partial s_{rz}}{\partial t} + v\frac{\partial s_{rz}}{\partial r} + u\frac{\partial s_{rz}}{\partial z} + \lambda s_{rz} = G\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right),
\end{aligned} \tag{5}$$

here $\lambda = 0$ for the elastic domain, for the plastic deformation is determined by the von Mises yield condition:

$$s_r^2 + s_z^2 + s_\varphi^2 + 2s_{rz}^2 = \frac{2}{3}Y_e^2 \qquad (6)$$

Y_e^2 is flow limit, $s_r, s_z, s_{\varphi,} s_{rz}$ are stress deviators, G is shear modulus. The flow limit Y_e depends on the temperature, pressure and other state parameters as:

$$Y_e = Y_0(1+\beta\varepsilon_u^p)^n(1-b\sigma(\frac{\rho_0}{\rho})^{1/3} - h(T-T_0)),$$

$$Y_0(1+\beta\varepsilon_u^p)^n \le Y_{\max},\ Y_0 = 0,\ \mathrm{T > T_m},$$

$$T_m = T_{m0}(\frac{\rho_0}{\rho})^{2/3}\exp(2\gamma_0(1-\frac{\rho_0}{\rho})), \qquad (7)$$

where $\varepsilon_u^p = \sqrt{2\varepsilon_{ij}^p\varepsilon_{ij}^p/3}$ is the intensity of plastic deformation tensor, T_m is the material melting temperature, $Y_0, Y_{\max}, T_{m0}, \beta, b, \gamma_0$ are materials constants, $\sigma_* = \sigma_*^0 Y/Y^0, \sigma_*^0$ are constants [3].

The problem of dynamic loading involves one to consider the system of motion together with the state equation that determines the relationship between pressure, density and one of the gas-dynamic variables - temperature or energy.

The equations of state of the medium allow closing the system of determining equations of motion, then select the numerical method for the developing of numerical solution algorithm.

The relevant equations of state have been chosen for description of the structural material, explosive substance, detonation products and ground.

State equation in the Tet's form for the different types of explosives (trinitrotoluene, cyclonite, TNT)

$$p = \frac{k}{n}((\frac{\rho}{\rho_0})^n - 1) \qquad (8)$$

where p is pressure, ρ is density, ρ_0 is initial density, k, n are constants.

State equation of such materials as aluminum, steel, copper, iron in the form of shock adiabat:

$$p = a_0^2\rho_0(1-\frac{\rho_0}{\rho})/((1-\frac{\rho_0}{\rho})\chi + 1)^2 \qquad (9)$$

where a, χ are constants.

Today there are complex models describing the explosion process. However, in the event of an explosion of industrial charges is possible to use a simplified gas explosion model.

State equation of the detonation products in the form of polytrope:

$$p = A\rho^\gamma$$

$$p = A\rho^{\beta_1} + B\rho^{\beta_2} \qquad (10)$$

here $A, \gamma, \beta_1, \beta_2$ are explosive substance constants.

State equation for non-metals and metals in the form of Mie-Gruneisen:

$$p = k_1(\frac{\rho}{\rho_0}-1) + k_2(\frac{\rho}{\rho_0}-1)^2 + k_3(\frac{\rho}{\rho_0}-1)^3 + \gamma_0 E \qquad (11)$$

where $k_1, k_2, k_3, k_4, \gamma_0$ are material constants.

State equation of the saturated ground that represented as three-component medium, consisting of solid skeleton, water and air.

Let us introduce following notation: d_i is a volume content of the i-th component, i=1, 2, 3. Here (i=1) corresponds to the air component, (i=2) – to the water component, and (i=3) to the solid component. Values d_i satisfy the condition $d_1 + d_2 + d_3 = 1$. If we denote initial ground density and pressure as ρ_0 and p_0 then the medium density for the initial pressure is $\rho_0 = \sum_{i=1}^{3} d_i\rho_i$:

$$\frac{\rho_0}{\rho} = \sum_{i=1}^{3} d_i\left[\frac{b_i(p-p_0)}{\rho_i c_i^2} + 1\right]^{-\frac{1}{b_i}} \qquad (12)$$

where b_i are isentropic exponents, c_i are sound velocities in these components for the case of initial pressure p_0 [2].

Seismic wave has generated as the function of pressure at the vertical boundary of the computational domain:

$$P(t) = (P_o e^{-\alpha t} + P_{oc})H(t) \qquad (13)$$

where $P_o + P_{oc} = P_{os}$ is peak impact pressure, P_{oc} is constant pressure, α is attenuation constant, $H(t)$ is unit step function [4].

III. NUMERICAL CALCULATIONS AND RESULTS

At the stage of computational experiment designing for solving such problems it is necessary to specify the initial and boundary conditions, as well as the values of various parameters and constants of materials for realistic equations of state.

At time t=0 μs the computational domain is a rectangular area in a two-dimensional Lagrangian coordinate system, where the initial state of relevant system components is simulated.

Shock wave impact on buried projectile is studied. Such projectiles may have different shape and size, generally consist of metal body filled with explosive, and may have one or more fuses.

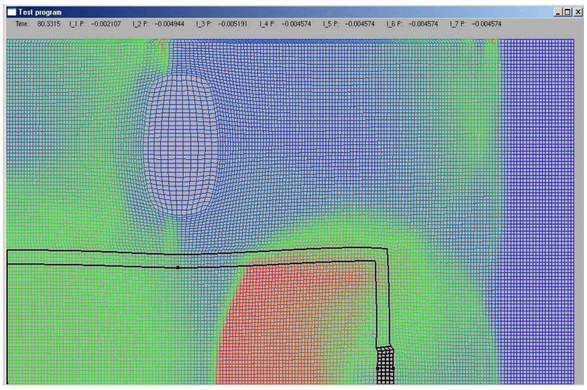

Fig. 1 The computational domain, variant 1

The projectile is modeled as aluminium construction (width is R, length is 3R, wall thickness is R/10), filled with explosive substance (TNT). In the end of the structure is the fuse (TNT with 4 % higher sensitivity than the filler).

There are several sensors on the inside of the tank shell. The sensors fix pressure, stress and other parameters at given time intervals.

The projectile is completely buried in the ground, is located at some distance from the ground surface, rigid walls are located right and left from it. The following numerical values of the constants are used in the calculations.

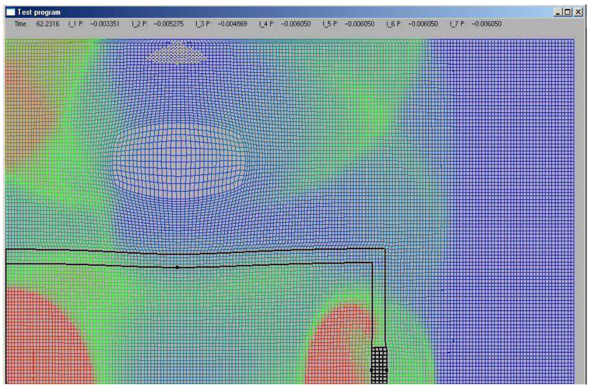

Fig. 2 The computational domain, variant 2

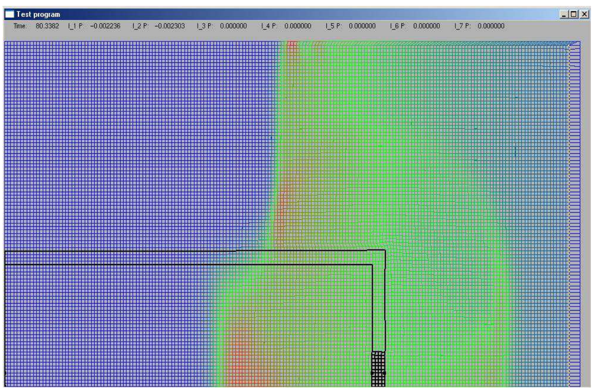

Fig. 3 The computational domain, variant 3-a

The whole construction is surrounded by the water-saturated ground, including 10 % of air, 30 % of water, 60 % of quartz.

We consider several variants of calculations.

Variant 1: The initiating ES is the rectangle with size R/4 x 3R/4 is detonated at time t=0 µs (Fig. 1).

Variant 2: The initiating ES with size 3R/4 x R/4 is at the distance R from the projectile shell and from the axis of symmetry OY (Fig. 2).

Detonation occurred inside the fuse in all variants of the calculation, but at different times: variant 1 - t=90 µs, variant 2 - t=75 µs. The detonation appeared inside the filler earlier than in the fuse in variant 1.

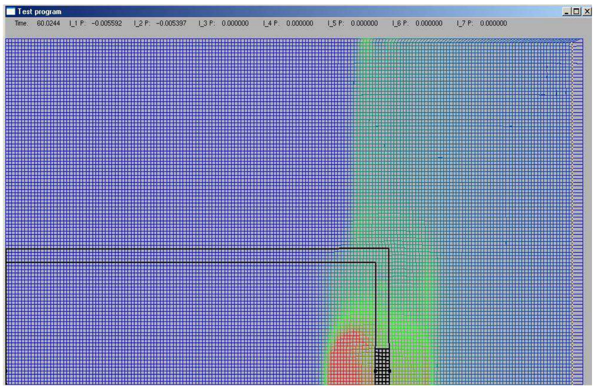

Fig. 4 The computational domain, variant 3-b

Two series of calculations for two values of the initial pressure $P_o = 17.0 \cdot 10^{-4}$ Mbar (variant 3-a) and $P_o = 19.2 \cdot 10^{-4}$ Mbar (variant 3-b) were conducted. The general computation time was t=180 µs.

The plane wave from the vertical boundary of the computational domain began propagation in time t=0 µs**,** reached the end of construction and partially reflected from it.

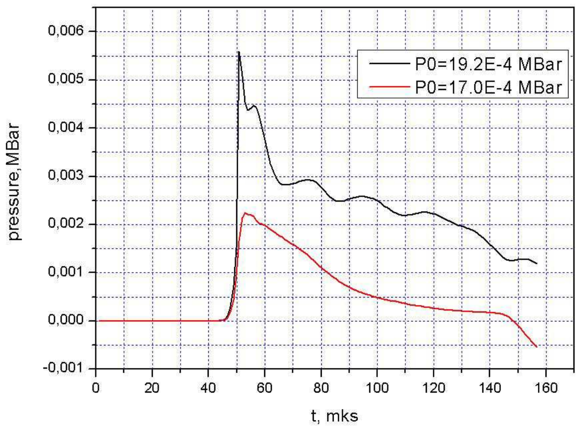

Fig. 5 Pressure distribution on the boundary "ground-fuse"

In both cases the shock wave came to the fuse in the same time (t=50 µs). The fuse did not blast out for the value of the initial pressure $P_o = 17.0 \cdot 10^{-4}$ Mbar, but it detonated in t=51 µs for $P_o = 19.2 \cdot 10^{-4}$ Mbar).

The numerical results show that minimum pressure to initiate detonation of fuse is P = 0.0054 Mbar.

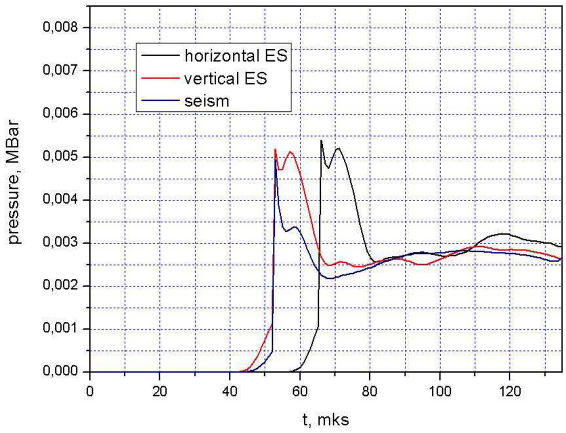

Fig. 6 Pressure distribution on the boundary "ground-fuse" for three variants

IV. CONCLUSION

In the paper presented examples of results of numerical calculations for solving of solid mechanics problems that considering physical effects and material properties.

The numerical analysis showing that for the given initial and boundary conditions projectile detonation primarily depends on the initial pressure value in the seismic waves function. The critical pressure to initiate fuse detonation is P = 0.0054 Mbar.

REFERENCES

[1] M. L. Wilkins. Modelling the behavior of materials. Struct. Impact and Grashworth. Proceeding of International Conference. V.2, London, New York, 1984.
[2] G. M. Lyakhov, G. I. Pokrovskii. Blast Waves in Soils. [in Russian]. Gosgortekhizdat, 1962.
[3] B. Rybakin. Computer Modeling of Dynamic Processes. CSJM, v.8, N 2(23), 2000, pp. 150-180.
[4] B. Rybakin, G. Secrieru, E. Gutuleac. Numerical analysis of reaction of buried charge to explosive or seismic loading**.** In: Proceedings of the International Conference on Intelligent Information Systems, 2013, Chisinau, pp. 148-151.