METHOD OF DETERMINING OF DISSIPATION ENERGY DURING THE MOVEMENT OF BINGHAM FLUID

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Abstract: This article discusses methods of determining the energy dissipation in the flow of Bingham fluids in channels of different shapes.
Keywords: Bingham fluid, the energy dissipation, screw machine, straight and flat channel, the algorithm calculated numbers.

1. INTRODUCTION

Thermal processes are the most common processes in food technologies [1]. Today lots of scientific paper describe heat transfer in Newtonian fluids [2-4]. But there are quite a number of food products (chocolate, pastry, confectionery) that relate to multicomponent systems having a similar structure and the nonlinear nature of the flow [5].

2. RECENT INVESTIGATIONS

For an assessment of the structural and mechanical properties, it is necessary to know the type of structure, the dependence of the viscosity of the shear rate, as well as the magnitude of dissipation [5].

It is known that during its movement non-Newtonian fluid loses some of the energy that goes into dissipative heat. Knowledge of the dissipation value allows to pick technological equipment with the best power reserve, thereby reducing production costs [6].

3. RESULTS OF RESERCH

This paper suggests a method for determining of the dissipation energy during the flow of Bingham fluids in channels of different shape.

Consider a straight and a flat channel. Splitting these channels on the cross sections with different expressions for the flow velocity are shown on Fig. The splitting elements are denoted as $S_y^{\pm}$, $S_x^{\pm}$ (see Fig.). In order to reduce records, various types of flows, correlated with corresponding subdomains of partitions should be written as:

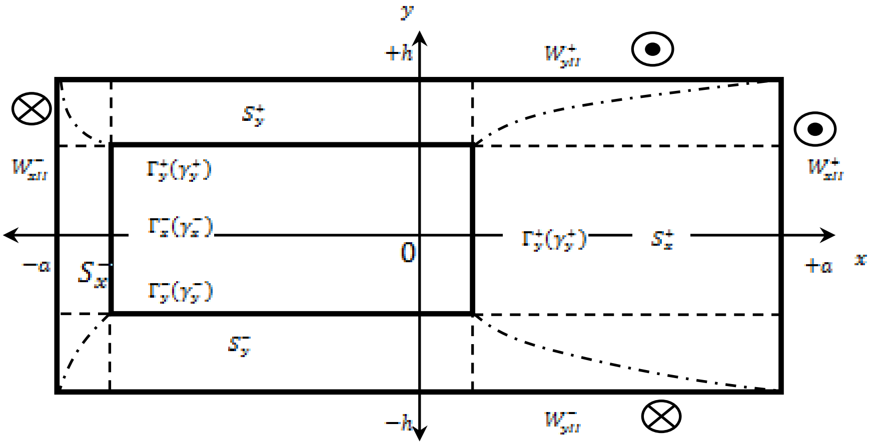

Fig. Partitioning rectangular channel into subregions

$$\upsilon_z^{\pm} = a_z^{\pm}\xi_y^2 + b_z^{\pm}\xi_y + c_z^{\pm} -$$

flat longitudinal flow in the $S^{\pm}$ (1)

$$\upsilon_z^{\pm} = a_z^{\pm}\xi_y^2 + b_z^{\pm}\xi_y + c_z^{\pm} -$$

flat longitudinal-transverse flow in $S^{\pm}$ (2)

$$\upsilon_y^{\pm} = a_y^{\pm}\xi_y^2 + b_y^{\pm}\xi_y + c_y^{\pm}$$

$$\upsilon_z^{\pm} = a_{zy}^{\pm}\xi_y^2 + b_{zy}^{\pm}\xi_y + c_{zy}^{\pm} -$$

rectangular longitudinal flow in $S_y^{\pm}$ and $S_x^{\pm}$ (3)

$$\upsilon_z^{\pm} = a_{zx}^{\pm}\xi_x^2 + b_{zx}^{\pm}\xi_x + c_{zx}^{\pm}$$

$$\upsilon_z^{\pm} = a_{zy}^{\pm}\xi_y^2 + b_{zy}^{\pm}\xi_y + c_{zy}^{\pm}$$

longitudinal component of rectangular flow in $S_y^{\pm}$ and $S_x^{\pm}$ (4)

$$\upsilon_z^{\pm} = a_{zx}^{\pm}\xi_x^2 + b_{zx}^{\pm}\xi_x + c_{zx}^{\pm}$$

$$\upsilon_x^{\pm} = a_{xx}^{\pm}\xi_x^2 + b_{xx}^{\pm}\xi_x + c_{xx}^{\pm}$$

transverse component of rectangular flow in $S_y^{\pm}$ and $S_x^{\pm}$ (5)

$$\upsilon_y^{\pm} = a_{yy}^{\pm}\xi_y^2 + b_{yy}^{\pm}\xi_y + c^{\pm}\xi_y$$

<u>flat longitudinal flow</u>

In the flat longitudinal flow the dissipation is generated only by a single В плоском продольном течении диссипацию порождает только одно summand $(\partial\upsilon_y/\partial_y)^2$. The magnitude of dissipation which affects the cross-section of flat channel equals to:

$$\dot{E} \equiv \iint \left(\frac{\partial \upsilon_z}{\partial_y} \right)^2 ds = \iint \left(\frac{\partial \upsilon_z^+}{\partial_y} \right)^2 ds_y^+ + \iint \left(\frac{\partial \upsilon_z^-}{\partial y} \right)^2 ds_y^- . \qquad (6)$$

Substituting the expressions for $\upsilon_z^{\pm}(y)$ from the formulas for $\upsilon_z^{\pm}$ we can come to the following integrals:

$$\frac{\dot{E}}{\mu} = \frac{1}{h} \int_{-1}^{\gamma-} d\xi_y \left(2a_z^- \xi_y + b_z^- \right)^2 + \frac{1}{h} \int_{\gamma+}^{1} d\xi_y \left(2a_z^+ \xi_y + b_z^+ \right)^2 . \qquad (7)$$

Final result for $\dot{E}/\mu$ value is as follows:

$$\frac{\dot{E}}{\mu} = \frac{1}{h} \left\{ \begin{aligned} & \left[\frac{4}{3}\left(a_z^+\right)^2 \left(1-\gamma^+\right)^3 - \frac{4}{3}\left(a_z^-\right)^2 \left(1+\gamma^-\right)^3 \right] + \left[2a_z^+ b_z^+ \left(1-\gamma^+\right)^2 + 2a_z^- b_z^- \left(1+\gamma^-\right)^2 \right] + \\ & \left[\left(b_z^+\right)^2 \left(1-\gamma^+\right) - \left(b_z^-\right)^2 \left(1+\gamma^-\right) \right] \end{aligned} \right\} \qquad (8)$$

<u>flat longitudinal-transverse flow</u>

IN this case there are two summands: $\left(\partial \upsilon_z^{\pm} / \partial y\right)^2$ and $\left(\partial \upsilon_y^{\pm} / \partial y\right)^2$. They are calculated the same way as for the flat longitudinal flow. Final result looks as follows:

$$\frac{\dot{E}}{\mu} = \frac{\dot{E}_z}{\mu} + \frac{\dot{E}_x}{\mu}; \qquad (9)$$

$$\frac{\dot{E}_z}{\mu} = \frac{1}{h} \left\{ \begin{aligned} & \left[\frac{4}{3}\left(a_z^+\right)^2 \left(1-\gamma_y^+\right)^3 - \frac{4}{3}\left(a_z^-\right)^2 \left(1+\gamma_y^-\right)^3 \right] + \left[2a_z^+ b_z^+ \left(1-\gamma_y^+\right)^2 + 2a_z^- b_z^- \left(1+\gamma_y^-\right)^2 \right] + \\ & \left[\left(b_z^+\right)^2 \left(1-\gamma_y^+\right) - \left(b_z^-\right)^2 \left(1+\gamma_y^-\right) \right] \end{aligned} \right\} \qquad (10)$$

$$\frac{\dot{E}_y}{\mu} = \frac{1}{h} \left\{ \begin{aligned} & \left[\frac{4}{3}\left(a_x^+\right)^2 \left(1-\gamma_{yx}^+\right)^3 - \frac{4}{3}\left(a_x^-\right)^2 \left(1+\gamma_{yx}^-\right)^3 \right] + \left[2a_x^+ b_x^+ \left(1-\gamma_{yx}^+\right)^2 + 2a_x^- b_x^- \left(1+\gamma_{yx}^-\right)^2 \right] + \\ & \left[\left(b_x^+\right)^2 \left(1-\gamma_{yx}^+\right) - \left(b_x^-\right)^2 \left(1+\gamma_{yx}^-\right) \right] \end{aligned} \right\} \qquad (11)$$

<u>rectangular longitudinal flow</u>

The magnitude of dissipation in this flow is the sum of four summands:

$$\frac{\dot{E}}{\mu} = \iint ds_y^+ \left(\frac{\partial \upsilon_z^+}{\partial y} \right)^2 + \iint ds_x^+ \left(\frac{\partial \upsilon_z^+}{\partial x} \right)^2 + \iint ds_x^- \left(\frac{\partial \upsilon_z^-}{\partial x} \right)^2 . \qquad (12)$$

Integrals of the derivatives values are recorded in the following way: for rectangular sections of each subdomain

$$\iint\left(\frac{\partial \upsilon_z^+}{\partial y}\right)^2 d\xi_y=\left[\frac{4}{3}\left(a_{zy}^+\right)^2\left(1-\gamma_y^+\right)^3+2a_{zy}^+b_{zy}^+\left(1-\gamma_y^+\right)+\left(b_{zy}^+\right)^2\left(1-\gamma_y^+\right)\right]\frac{1}{h^2};\text{ in } S_y^+ \qquad (13)$$

$$\iint\left(\frac{\partial \upsilon_z^-}{\partial y}\right)^2 d\xi_y=\left[-\frac{4}{3}\left(a_{zy}^-\right)^2\left(1+\gamma_y^-\right)^3+2a_{zy}^-b_{zy}^-\left(1+\gamma_y^-\right)-\left(b_{zy}^-\right)^2\left(1+\gamma_y^-\right)\right]\frac{1}{h^2};\text{ in } S_y^- \qquad (14)$$

$$\iint\left(\frac{\partial \upsilon_z^+}{\partial x}\right)^2 d\xi_x=\left[\frac{4}{3}\left(a_{zx}^+\right)^2\left(1-\gamma_x^+\right)^3+2a_{zx}^+b_{zx}^+\left(1-\gamma_x^+\right)+\left(b_{zx}^+\right)^2\left(1-\gamma_x^+\right)\right]\frac{1}{a^2};\text{ in } S_x^+ \qquad (15)$$

$$\iint\left(\frac{\partial \upsilon_z^-}{\partial x}\right)^2 d\xi_x=\left[-\frac{4}{3}\left(a_{zx}^-\right)^2\left(1+\gamma_x^-\right)^3+2a_{zx}^-b_{zx}^-\left(1+\gamma_x^-\right)-\left(b_{zx}^-\right)^2\left(1+\gamma_x^-\right)\right]\frac{1}{a^2};\text{ in } S_x^-. \qquad (16)$$

Each of these expressions should be integrated in the corresponding subdomain $S_y^{\pm}$, $S_x^{\pm}$. Each of the subdomains consists of two curvilinear triangles and one rectangle. Curvilinear triangles have the line that splits neighboring subdomains as their “curvilinear”.

Thus each integral can be expressed as the sum of three integrals of the following form:

$$\iint ds_y^+\left(\frac{\partial \upsilon_z^+}{\partial y}\right)^2=h^2\int_{\gamma_y^+}^{1}d\xi_y\left(\frac{\partial \upsilon_z^+}{\partial \xi_y}\right)^2\times\left(\gamma_x^+-\gamma_x^-\right)+ah\int_{-1^+}^{\theta^+(\xi_x)}d\xi_y\int_{-1}^{\gamma_x^+}d\xi_x\left(\frac{\partial \upsilon_z^+}{\partial \xi_y}\right)^2+$$
$$+ah\int_{\varepsilon^+(\xi_x)}^{1}d\xi_y\int_{\gamma_x^+}^{1}d\xi_x\left(\frac{\partial \upsilon_z^+}{\partial \xi_y}\right)^2 \qquad (17)$$

$$\iint ds_y^-\left(\frac{\partial \upsilon_z^-}{\partial y}\right)^2=h^2\int_{-1}^{\gamma_y^-}d\xi_y\left(\frac{\partial \upsilon_z^-}{\partial \xi_y}\right)^2\times\left(\gamma_x^+-\gamma_x^-\right)+ah\int_{-1}^{\varepsilon^-(\xi_x)}d\xi_y\int_{\gamma_x^+}^{1}d\xi_x\left(\frac{\partial \upsilon_z^-}{\partial \xi_y}\right)^2+$$
$$+ah\int_{-1}^{\theta(\xi_x)}d\xi_y\int_{-1}^{\gamma_x^-}d\xi_x\left(\frac{\partial \upsilon_z^-}{\partial \xi_y}\right)^2 \qquad (18)$$

$$\iint ds_x^+\left(\frac{\partial \upsilon_z^+}{\partial x}\right)^2=a^2\int_{-1}^{\gamma_x^-}d\xi_x\left(\frac{\partial \upsilon_z^+}{\partial \xi_x}\right)^2\times\left(\gamma_y^+-\gamma_y^-\right)+ah\int_{-1}^{\left[o^+\right]^{-1}(\xi_x)}d\xi_x\int_{\gamma_y^+}^{-1}d\xi_y\left(\frac{\partial \upsilon_z^+}{\partial \xi_x}\right)^2+$$
$$+ah\int_{\left[\theta^+\right]^{-1}}^{-1}d\xi_x\int_{(\xi_y)\gamma_y^+}^{1}d\xi_y\left(\frac{\partial \upsilon_z^+}{\partial \xi_x}\right)^2 \qquad (19)$$

$$\iint ds_x^- \left(\frac{\partial \upsilon_z^-}{\partial x}\right)^2 = a^2 \int_{-1}^{\gamma_x^-} d\xi_x \left(\frac{\partial \upsilon_z^-}{\partial \xi_x}\right)^2 \times \left(\gamma_y^+ - \gamma_y^-\right) + ah \int_{\left[\varepsilon^-\right]^{-1}}^{1} d\xi_x \int_{\left(\xi_y\right)^{-1}}^{\gamma_y^-} d\xi_y \left(\frac{\partial \upsilon_z^-}{\partial \xi_x}\right)^2 + $$
$$+ ah \int_{-1}^{\left[\theta^-\right]^{-1}} d\xi_x \int_{-1}^{\left(\xi_y\right)_{\gamma_y^-}} d\xi_y \left(\frac{\partial \upsilon_z^-}{\partial \xi_x}\right)^2 \qquad (20)$$

$$\left(\frac{\partial \upsilon_z^\pm}{\partial \xi_y}\right) = \left(2a_{zy}^\pm \xi_y + b_{zy}^\pm\right)^2; \quad \left(\frac{\partial \upsilon_z^\pm}{\partial \xi_x}\right)^2 = \left(2a_{zx}^\pm \xi_x + b^\pm\right)^2. \qquad (21)$$

In these formulas $\theta^\pm\left(\xi_y\right)$, $\varepsilon^\pm\left(\xi_y\right)$ – expressions for splitting lines between subdomains of the partition of the rectangular cross-section of the channel; $\left[\theta^\pm\right]^{-1}$, $\left[\varepsilon^\pm\right]^{-1}$ – inverse functions to $\theta^\pm$ and $\varepsilon^\pm$.

4. CONCLUSION

The same algorithm can be used to calculate dissipation energy of generalized shear fluid. The mandatory phases of these calculations are the partitioning of rectangular cross-section of the straight channel and calculation of the integrals from velocity derivatives. But, while for Bingham fluid the only the part of cross-section of the channel contributes to the dissipation energy, for the generalized shear fluid has fluidity in the entire cross-section of the channel. While cubical polynomials originate during integration of velocity derivatives and the integration itself is performed quite easily, certain transformations should be performed during integration of derivatives of generalized-shear fluid velocity. The technique of these transformations will be discussed in subsequent papers.

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