Empirical Linearly- Hyperbolic Approximation of the *I-V* Characteristic of the *p–n* Junction Devices

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***Abstract* — Empirical linearly-hyperbolic approximation of the *I-V* characteristic of diodes and solar cell is presented. This approximation is based on hyperbolic dependence of a current of the *p–n* junctions on voltage for large currents. Such empirical approximation is compared with the early proposed formal linearly-hyperbolic approximation of a solar cell. There are obtained the expressions, defining laws of change of parameters of formal approximation at variation of a photocurrent for the set of characteristics. It allows simplifying a finding of parameters of approximation on actual curves, to specify their values.**

Analytical calculation of the load regime for linearly - hyperbolic model leads to quadratic equation. Also, this model allows to define soundly a deviation from the maximum power regime and to compare efficiency of regimes of solar cells with different parameters.

***Index Terms* — analytical calculation, approximation, *I-V* characteristic, *p–n* junction, solar cell generator.**

I. INTRODUCTION

For analysis and calculation of power supply systems based on solar cells generators mathematical model of cell is necessary. The traditional exponential model of *p–n* junctions or of I-V characteristic [1] requires the iterative numerical calculation methods. Model in the form of the fractional-quadratic expression leads to the cubic equation of the circuit with the resistance load [2]. It complicates the calculations, especially, in the real time regime.

More convenient model represents the linearly - hyperbolic expression [3]. The calculation with the resistance load leads to the quadratic equation. Also, this model makes it possible to soundly determine deviation from the maximum power regime [4]. The model parameters are calculated for the concrete curve (with the appropriate level of insulation) of the set of characteristics from two points near the point of maximum power. But values of the calculated parameters depend on coordinates of these points, which complicate the finding of dependence of parameters on photocurrent level. Therefore, there is a problem of a physical validation of linearly - hyperbolic approximation and obtaining of dependence of the model parameters on photocurrent level [5].

II. FEATURES OF KNOWN MODELS OF *I - V* CHARACTERISTIC OF THE *p – n* JUNCTION DIODE

The exponential expression of *I-V* characteristic widely utilized in the theory of semiconductor devices [5]:

$$i = I_0\left[\exp\left(\frac{u}{\varphi_T}\right) - 1\right], \qquad (1)$$

where I_0 is the saturation current or scale current of thediode, φ_T is the thermal voltage, u is the applied or external voltage.

The expression of reverse characteristic:

$$u = \varphi_T\, Ln\left(\frac{i}{I_0} + 1\right) \qquad (2)$$

The resistance of diode or of junction to direct current:

$$R = \frac{u}{i} = \frac{\varphi_T}{i} Ln\left(\frac{i}{I_0} + 1\right)$$

At the zero point this resistance is equal to:

$$R(0) = \frac{\varphi_T}{I_0} \qquad (3)$$

The application of diodes in the power electronics is connected with the large forward currents. Therefore, it is necessary to take into account the voltage drop across the base resistance r_B. Therefore, this characteristic can be named linear -exponential.

Also, for greater current and growing injection level the modulation of the base resistance reveals itself. To take into account this effect, parameter $m = 1.5 - 2$ is introduced:

$$i = I_0 \left[\exp\left(\frac{u - ir_{\text{Б}}}{m\varphi_T} \right) - 1 \right], \tag{4}$$

$$u = m\varphi_T \, Ln\left(\frac{i}{I_0} + 1 \right) + ir_{\text{Б}} \tag{5}$$

But a more strict analysis shows that for the greater currents and high levels of injection, the elementary exponential dependence of current on voltage (1) is not valid essentially already. Let us consider the approximations of the forward characteristics of the power diode according to expression (4) on Fig.1. It is possible to notice that comprehensible accuracy of approximation obtained with parameters m, I_0 which are **considerably different** from the recommended and actual values.

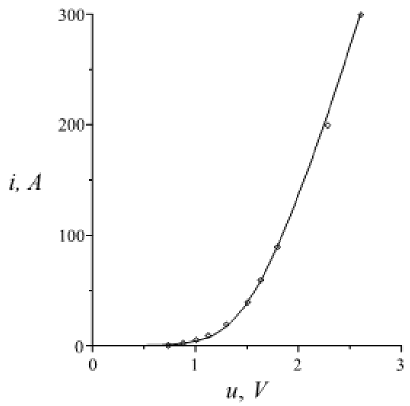

Fig.1. The exponential approximation of the forward *I-V* characteristic of *HFA70NH60* diode:
line- $m\varphi_T = 0.18V$, $r_B = 0.0028\,\Omega$, $I_0 = 0.017\,A$;
boxes- actual values

III. Connection of parameters of linearly-hyperbolic approximation with the parameters of exponential model I-V characteristic of diode

For exponential model of the idealized diode, at approach of forward bias voltage to the built-in potential V_{bi} the forward current sharply grows and the saturation current, in turn, does not depend on reverse voltage. Such behavior of currents allows applying the hyperbolic function presented on Fig.2. In this case, asymptotes are the voltage $u = \varphi_K$ and current $i = -aI_0$, where a is the scale factor. Such hyperbolic function is described by the following expression:

$$i = aI_0 \frac{u}{\varphi_K - u} \tag{6}$$

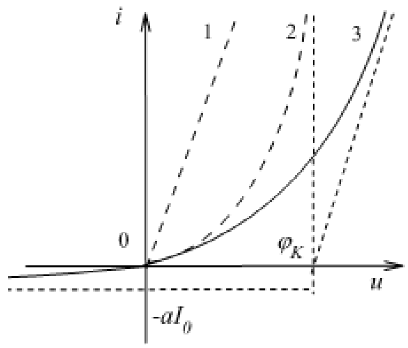

Fig.2. The proposed approximation of the forward I-V characteristic of diode: 1- line component, 2- hyper component, 3- resulting linearly-hyperbolic characteristic

Reverse expression looks like:

$$u = \varphi_K \frac{i}{aI_0 + i} \tag{7}$$

Then the nonlinear component of resistance to a direct current takes the form:

$$R(i) = \frac{u}{i} = \frac{\varphi_K}{aI_0 + i}.$$

For zero values of a current and voltage the value of resistance is equal:

$$R(0) = \frac{u}{i} = \frac{\varphi_K}{aI_0} \tag{8}$$

Equating expression (8) with similar expression (3) we receive the parameter a :

$$a = \frac{\varphi_K}{\varphi_T}$$

Definitively, we receive expressions of the hyperbolic characteristic correct for the forward branch:

$$i = I_0 \frac{\varphi_K}{\varphi_T} \frac{u}{\varphi_K - u} \tag{9}$$

$$u = \varphi_K \frac{i}{\frac{\varphi_K}{\varphi_T} I_0 + i} \tag{10}$$

The received expressions also represent the idealized model of the diode. It is possible to accept that for $u \to \varphi_K$ the forward current is limited by a linear part of resistance of junction from base r_{B1} and by resistance of the base neutral region r_{B2}. Therefore, expression (10) is naturally supplemented by a linear component as well as expression (5):

$$u = \varphi_K \frac{i}{\frac{\varphi_K}{\varphi_T} I_0 + i} + ir_B, \tag{11}$$

where $r_B = r_{B1} + r_{B2}$.

Thus, empirical expression of the characteristic is received. This implies the equivalent scheme of the diode presented on Fig. 3. The diode is replaced with consecutive connection of nonlinear and linear resistance (or of conductance).

From expression (11) we receive the direct expression of characteristic:

$$i^2 r_B + i(\varphi_K - u - r_B I_0 \varphi_K / \varphi_T) - u I_0 \varphi_K / \varphi_T = 0$$

The current is calculated as the solution of quadratic equation for set voltage. It **essentially differs** from the transcendental exponential expressions (4).

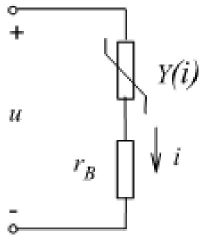

Fig.3. The equivalent scheme of the diode in linearly-hyperbolic approximation

The approximation of the power diode, corresponding to expression (11), is presented on Fig.4.

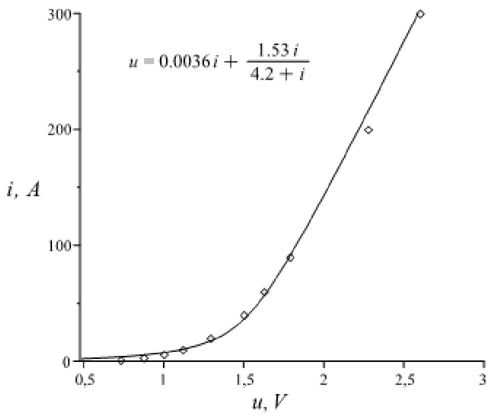

Fig.4. The linearly-hyperbolic approximation of HFA70NH60 diode

It is obviously that the offered linearly-hyperbolic approximation quite satisfies to practical application

IV. THE CONNECTION OF PARAMETERS OF EMPIRICAL AND FORMAL LINEARLY - HYPERBOLIC APPROXIMATION OF A SOLAR BATTERY CHARACTERISTIC.

Let us give the expression of a solar cell characteristic:

$$u = \varphi_T \, Ln\left(\frac{I_{Ph} - i}{I_0} + 1\right) - i\, r_S , \qquad (12)$$

where r_S is a resistance of losses, I_{Ph} is a photocurrent. The light characteristic of a solar cell is presented on Fig.5. Current $I_{Ph} = 0$ for dark characteristic. Therefore, the dark characteristic represents simply the diode characteristic.

This implies that the working region of a solar cell characteristic corresponds to the forward characteristic of the diode displaced on an axis of currents on value I_{Ph}.

Therefore, it is possible to use the offered linearly-hyperbolic approximation of a forward characteristic of the diode for approximation of a working region of a solar cell characteristic.

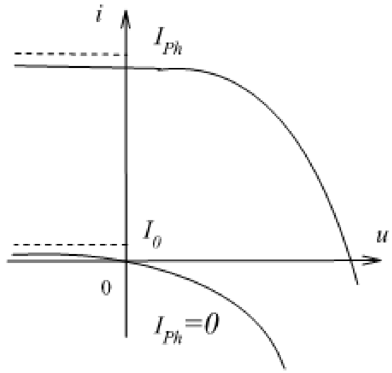

Fig.5. Light characteristic of a solar cell

On this basis, it is possible to present the equivalent linearly-hyperbolic scheme of a solar cell on Fig.6.

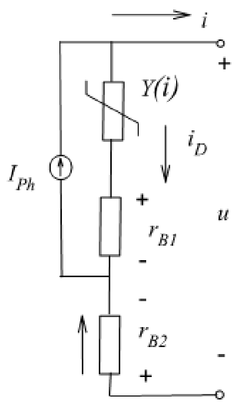

Fig.6. Equivalent linearly-hyperbolic scheme of a solar cell

Taking into account the specified directions of currents and polarity of voltage of the scheme elements, we receive the following relations:

$i_D = I_{Ph} - i$, $u = u_D + u_{B1} - u_{B2} = v_1 + v_2$.

Using expression of the characteristic of the diode (11), we find the load voltage components:

$$v_1 = \frac{\varphi_K I_{Ph}}{I_{Ph} + I_0 \varphi_K / \varphi_T} \cdot \frac{(1 - i / I_{Ph})}{1 - \dfrac{i}{(1 + I_0 \varphi_K / I_{Ph} \varphi_T) I_{Ph}}} ,$$

$$v_2 = I_{Ph} \, r_{B1} \left(1 - i \frac{r_{B1} + r_{B2}}{I_{Ph} \, r_{B1}}\right)$$

Let us present the obtained early (expression (4) in [4]) of a formal approximation of the solar cell characteristic:

$$u(i) = u_1(i) + u_2(i) = E_1 \frac{1 - i/I}{1 - i/AI} + E_2(1 - i/I)$$

The plot of this expression is presented on Fig.7 where a hyperbolic $u_1(i)$, linear $u_2(i)$ component of voltage and short circuit current I are selected.

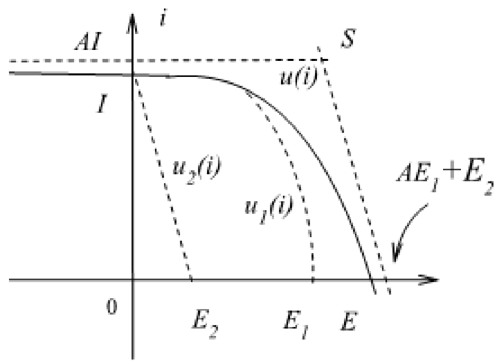

Fig.7. The plot of a formal approximation of the solar cell characteristic

The parameter A sets the degree of curvature of a hyperbole. If we compare this expression with the offered empirical expression, it is possible to determine the following:

$$E_2 = I_{Ph} r_{B1},\ I \cong I_{Ph},\ r_{B2} < r_{B1},$$

$$E_1 = \frac{\varphi_K I_{Ph}}{I_{Ph} + I_0 \varphi_K / \varphi_T}, \qquad (13)$$

$$A = 1 + I_0 \varphi_K / I_{Ph} \varphi_T \geq 1 \qquad (14)$$

From expression (13) follows that the voltage $E_1 \to \varphi_K$ while the current I_{Ph} is increasing. And voltage E_1 is proportional to a current I_{Ph} for small values of this current, it is corresponds to physical sense.

From expression (14) it also follows that parameter A does not depend significantly on current I_{Ph} for enough great values of this current and parameter A is decreased while current I_{Ph} is increased.

From expressions (13, 14) it turns out that $E_1 A = \varphi_K$. This implies that at processing of actual curves and calculation of parameters of formal approximation the relation $E_1 A \cong const$ should be carried out. It is confirmed also by the date [4]. In a practical example of solar cell MSX120, the constant value of parameter $A = 1.004$ for different insulation levels is quite soundly accepted.

As it has already been specified, the results of parameter calculations of formal approximation strongly depend on a choice of points near to the maximum power point. Therefore, the relations (13, 14) allow reducing this uncertainty.

V. CONCLUSION

The received expressions define the rules of change of parameters of formal approximation when parameters of family of curves of a solar cell characteristic are changed. It allows simplifying a method of finding of parameters of approximation on actual curves, to specify their values.

The suggested approach can be applied to the description of *I-V* characteristics of bipolar transistors.

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