CALCULATION OF FREEZING OF A CYLINDRICAL SURFACE

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In cooling technology the freezing process plays one of the main roles in the technical chain. Knowing the kinetics of the freezing process, allow us to perform it under the most optimal regime. A special interest shows the separation process of liquid products under low temperatures, when the moisture is freezing, there appears an augmentation of dry matter in the remaining solution. The low–temperature separation processes are based on using different freezing installation of different construction. The most interesting are those that support directed water crystallization onto cooling surface. Creating modular freezers presumes a calculation method for them, and running the whole process just in one module makes it ineffective. From this point of view process' modeling is very welcomed.

Let's analyze water from a solvent freezing process occurring in normal convection conditions:

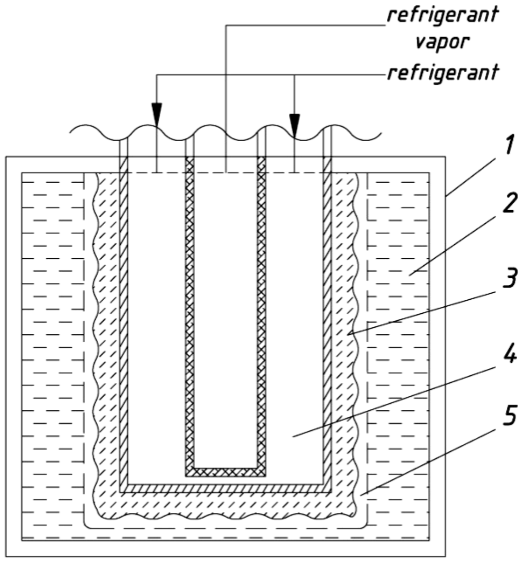

Fig. 1 Water from solvent freezing process
1 – isolated cylindrical capacity; 2 – freezing out solvent; 3 – ice block;
4 – freezing machine evaporator; 5 – diffusive boundary layer.

Freezing water from a solvent process has place in two stages.

First one – the solvent is cooled to the initial cryoscopic temperature (this can happen inside the freezer itself or in the cooler). On the second stage, on its exterior is forming a layer of ice, because of freezing agent (4) boiling, which is thickening as time passes, as well the concentration of dry material in the remaining solvent is increasing. As ice layer is thickening it increases the thermal resistance which lowers freezing machine efficiency. This process is maintaining until when the remaining solvent achieves necessary dry material percentage. Sometimes, because of ice's layer high thermal resistance is required its removal until stipulated concentration arrives, or to move formed thick solvent into another block, e.g. start the process from the beginning but with a higher origin concentration of dry material.

From this point of view it is necessary to calculate the thickness of frozen ice, temperature's distribution inside the ice layer (3) and inside the heat boundary layer (5).

To solve the heat problem, that is complicated because of the phase passage, we'll assume that: thermo physical coefficients are constant, the phase transfer has place at ice – liquid boundary.

Our case energy equation could be written excluding lateral thermal perturbations, if it happens to have a symmetrical crystallization, in this form:

$$\frac{\partial T_1}{\partial \tau} = \alpha_1 \left(\frac{\partial}{\partial R} R \frac{\partial T}{\partial R} \right) \qquad R_0 \leq R \leq \xi \qquad (1)$$

With lateral conditions

$$T_1(R_0 \tau) \qquad \text{when } R = R_0 \qquad (2)$$

$$T_1(R_0 \tau) \qquad \text{when } R = \xi \qquad (3)$$

Energy equation for the boundary layer can write like this:

$$\frac{\partial T_2}{\partial \tau} + V_R \frac{\partial T}{\partial R} + V_2 \frac{\partial T}{\partial_2} = \alpha_2 \frac{\partial}{\partial R}\left(R \frac{\partial T}{\partial R} \right) + \alpha_2 \frac{\partial^2 T}{\partial z^2} \qquad (4)$$

Boundary conditions for boundary layer will be written in the next form:

$$\left. \frac{\partial T}{\partial R} \right|_{R=3} = -\frac{\alpha}{\lambda_2} \left(T_\xi - T_c \right) \qquad (5)$$

$$T_2(\xi, \tau) = T_{cr} \qquad \text{when } R = \xi \qquad (6)$$

$$\lambda_1 \frac{dT_1}{dR} = \lambda_2 \frac{dT_2}{dR} + 4\rho \frac{d\xi}{d\tau} \qquad \text{when } R = \xi \qquad (7)$$

Equation's solution for an unlimited hollow cylinder will have the form:

$$T_1(R_1\tau) = \\ = \frac{1}{\ln\frac{3}{R_0}}\left[T_0 \ln\frac{\xi}{R_0} + T_{cr}\ln\frac{R}{R_0}\right] + \\ + \sum_{n=1}^{\infty}\frac{V_0\left(\mu\frac{R}{R_0}\right)exp(\mu_n^2 F_0)}{I_0^2(\mu_n) - I_0^2\left(\mu_n\frac{R}{R_0}\right)}\left\{\frac{\pi^2}{2R_0^2}\mu_n^2 I_0\left(\mu_n\frac{R}{R_0}\right)\int_{R_0}^{R} R\,f(R)V_0\left(\mu_n\frac{R}{R_0}\right)dR - \\ - \pi I_0(\mu_n m)\left[T_2 I_0 \mu_n T_{cr} I_0\left(\frac{R_2}{R_0}\mu_n\right)\right]\right\} \quad (8)$$

The roots of μ_n are defined from the specific equation:

$$I_0(\mu)Y_0\left(\frac{R}{R_0}\mu\right) - I_0\left(\frac{R}{R_0}\mu\right)Y_0(\mu) \quad (9)$$

$$Y_0\left(K\frac{R}{R_0}\right) = \frac{I_0(KR_0)Y_0(KR)}{I_0(KR)}$$

where
$$\int_{R_0}^{\xi} RV_0^2(K_nR)dR = \frac{2[I_0^2(K_nR_0) - I_0^2(K_nR)]}{\pi^2 K_n^2 I_0^2(K_nR)}$$

$$\mu_n = K_nR_0; \quad F_0 = \frac{a\tau}{R_0^2}$$

where: α – thermal diffusivity coefficient, [m^2/s];
τ – time, [s];
R_0 – freezing machine evaporator radius, [m],
Index *1* – refers to the frozen layer;
Index *2* – refers to the boundary liquid phase;
ξ – phase transfer zone coordinate;
$\lambda_{1,2}$ – thermal conductivity coefficient of frozen and boundary layers, [W/m·K];
L – crystallization latent heat, [J/kg];
c – specific heat, [J/(kg·K)].

Equation (4) resolution, describing temperature's distribution in liquid boundary layer, could be simplify by tacking the radius $R + \delta \to \infty$, which means solving a simple (сугубо плоскую) equation. The resolution in this case could be shown like this [2]:

$$\frac{T_2(R_1\tau) - T_{in}}{T_m - T_{in}} = \\ = 1 - \\ - \sum_{n=1}^{\infty} A_n \left\{ I_0(\mu_n) \cos\left[\mu_n K_a^{1/2}\left(\frac{R}{\xi} - 1\right)\right] - \\ - K_\xi I_1(\mu_n) \sin\left[\mu_n K_a^{1/2}\left(\frac{R}{\xi} - 1\right)\right]\right\} exp(-\mu_n^2 F_0) \tag{10}$$

where: μ_n – radicals of the characteristic equation;

$$I_0(\mu)\left[Bi cos\, K_a^{1/2}(K_R - 1)\mu - K_a^{1/2} K_R \mu \sin K_a^{1/2}(K_R - 1)\mu\right] - \\ - K_\xi I_1(\mu)\left[Bi \sin K_a^{1/2}(K_R - 1)\mu + K_a^{1/2} K_R \mu \cos K_a^{1/2}(K_R - 1)\mu\right] = 0$$

The coefficient

$$A_n = \\ = \frac{2BiK_\xi\left[K_a^{1/2}(K_R - 1)\mu_n + Bi\, tan\, K_a^{1/2}(K_R - 1)\mu_n\right]}{\mu_n I_0(\mu_n) \sin K_a^{1/2}(K_R - 1)\mu_n} + \\ + \left\{\left[K_\xi^2 K_a (K_R - 1)^2 \mu_n^2 + Bi^2\right] \cdot cot\, K_a^{1/2}(K_R - 1)\mu_n + \frac{2K_\xi K_a^{1/2}(K_R - 1)}{sin\, 2K_a^{1/2}(K_R - 1)\mu_n} \cdot \right. \\ \cdot \left[Bi^2 + K_a(K_R - 1)^2 {\mu_n}^2\right] + \\ + \left[K_a (K_R - 1)^2 {\mu_n}^2 + 2K_\xi K_a^{1/2}(K_R - 1) \cdot Bi + \right. \\ \left. + K_\xi^2 Bi^2\right] tan\, K_a^{1/2}(K_R - 1)\mu_n + K_\xi\, K_a (K_R - 1)^2 \mu_n^2 + \\ \left. + 2K_\xi^2 K_a^{1/2}(K_R - 1)\mu_n Bi - 2K_a^{1/2}(K_R - 1)\mu_n Bi - \frac{K_\xi Bi^2}{\mu_n}\right\}^{-1}$$

$$K_a = \frac{a_1}{a_2}; \quad K_R = \frac{R+\delta}{\xi}; \quad K_\xi = \sqrt{\frac{\lambda_1 c_1 \rho_1}{\lambda_2 c_2 \rho_2}};$$

In extreme cases when $Bi \to \infty$:

$$\frac{T_2(R_1\tau) - T_0}{T \quad - T_0} = \\ = 1 - \\ - \sum_{n=1}^{\infty} \frac{2\,sin\left[K_a^{1/2}\left(K_R - \frac{R}{\xi}\right)\mu_n\right] exp(-\mu_n^2 F_0)}{\mu_n\left[\frac{K_\xi^2 - 1}{K_\xi} sin^2\,K_a^{1/2}\,(K_R - 1)\mu_n - \frac{1}{2\mu_n}\right] sin\,2K_a^{1/2}(K_R - 1)\mu_n + b} \tag{11}$$

where $b = sin^2\,K_a^{1/2}\,(K_R - 1) + {1}/{K}$

To obtain the law of distribution of freezing distribution, we put the obtained results from equations (8) and (11) to equation (7). At the established regime, when the liquid product is cooling lower then 4°C, then for an engineering calculus is enough to consider a quasi – stationary problem, i.e.:

$$T_1 = \frac{(T_{cr} - T_0)\,ln\,R}{ln\,\frac{\xi}{R_0}} + T_0\,ln\frac{R_0}{\xi} \qquad R_0 \le R \le \xi \tag{12}$$

Temperature distribution in boundary layer will be represented in following way:

$$T_2 = \frac{T_{cr}\,ln\,R}{\left(ln\,\xi + \frac{V_R \xi}{a}\right)} + \frac{T_{cr} V_R \cdot R}{a} \tag{13}$$

$$T_2 = \frac{(T_{cr} - T_m)}{2\sqrt{a_2 \tau}} + T_{cr}$$

The velocity of ice deposing on the surface of the evaporator, will be written like:

$$\frac{d\xi}{d\tau} = \frac{1}{L\rho}\left[\frac{\lambda_1(T_{cr} - T_0)}{ln\frac{\xi}{R_0}\cdot\xi} - \frac{\lambda_2 T_{cr}}{2\sqrt{a\tau}}\right] \tag{14}$$

By knowing the freezing velocity and by identifying the thickness, one can find the performance of freezing machine.

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