GENERALIZATION OF THE THEVENIN AND NORTON EQUIVALENT GENERATORS. PROJECTIVE GEOMETRY METHOD

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Abstract. *The approach on the basis of projective geometry is used for interpretation of changes or "kinematics" of regimes of the circuit with changeable loads. The generalized equivalent generators of the active network in a form of the passive circuit and a set of the voltage and current sources are proposed. The parameters of these sources do not depend on certain conductivities of the passive circuit.*

Key-Words: *Thevenin's and Norton's theorems, load characteristics, projective coordinates, active networks.*

I. Introduction

The method of the equivalent generator, e.g. Thevenin's theorem, represents an active two-pole as a voltage source with an internal resistance [1]. It is known, also, an alternative to this generator [2]. However, if any resistance is changed in an active two-pole, the open circuit voltage is changed also. Therefore, it is not convenient to use this voltage as the parameter of the equivalent generator. Also, the method of the equivalent generator represents the active two-port network as the passive two-port network and the separated sources of the voltage or current at the terminals of two loads [1]. The parameters of these sources correspond to the open circuit voltage or the short circuit current of both loads. But, if any resistance of the passive two-port network is changed also, the recalculation of the values of these sources of the voltage or the current is necessary. In a number of articles of the author, the approach is developed for interpretation of changes or "kinematics" of circuit regimes on the basis of projective geometry. It appears that the external characteristic is transformed into a bunch of the straight lines for various values of the internal resistance of this two- pole [3]. Since the coordinates of the center of a bunch do not depend on this changeable element, they can be accepted as the parameters of the generalized equivalent generator of two- and multi- ports [4], [5]. The basic results are discussed further.

II. Generalized equivalent generator of the active two-pole with changeable resistance

It is known (Thevenin's theorem), any linear circuit (an active two-pole) A relative to load terminals R_H is replaced by a voltage source U_0 in series with a resistance R_i as shown in Fig.1.

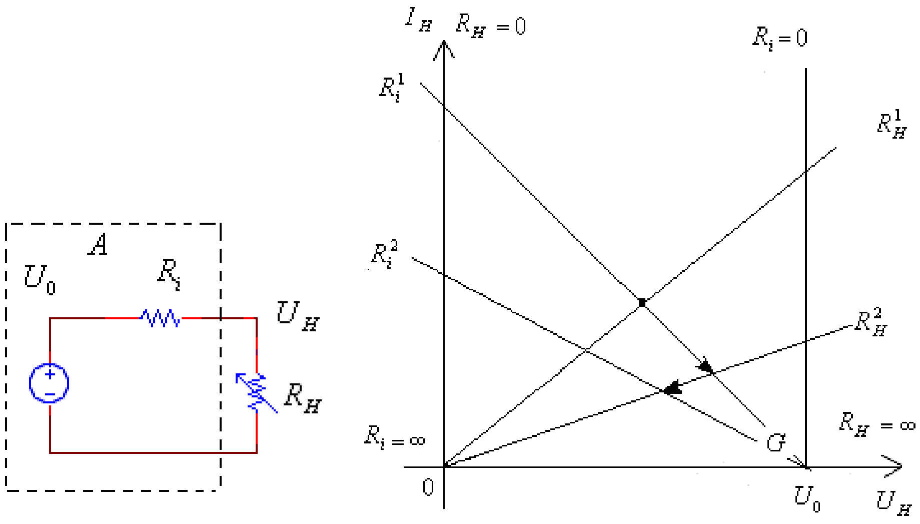


Fig.1. Active two-pole A and its $I-U$ characteristic family

Let two cases of change of elements R_H, R_i be considered.

Case1. $R_i = const$. A load straight line or the $I-U$ characteristic $I_H(U_H)$ is shown in Fig.1

$$I_H = \frac{U_0}{R_i} - \frac{U_H}{R_i}.$$

A bunch of straight lines with the parameter R_H and the centre in a point 0 corresponds to this straight line

$$I_H = \frac{1}{R_H} U_H \;.$$

It is obviously, that values $R_H = 0, R_H = \infty$ determinate characteristic regimes as short and open circuits. The point $R_H = R_i$ is a scale or unit one. The regime change $R_H^1 \to R_H^2$ can be expressed by the cross ratio of four points relatively the characteristic points [3].

Case2. Now, let the resistor R_i be changed, $R_i^1 \to R_i^2$. In this case, the $I-U$ characteristic family or a bunch of straight lines R_i is obtained with the center G in Fig.1. The coordinate of the center G corresponding U_0 does not depend on resistor R_i. Physically, it means that the current through this element is equal to zero. The element R_i can accept such characteristic values as $R_i = 0, R_i = \infty$. The third characteristic value is not present for R_i.

Let an active two-pole A with changeable resistance R_2 be considered and load $R_H = R_1$ with voltage $U_H = U_1$ in Fig. 2. Setting values R_2, we can receive expression $I_1 = I_1(U_1, R_2)$ for a bunch of the load straight lines with the center G. The position of the center G (in the second or the fourth quadrant) is corresponded by a kind of the energy source of the two-pole. If it is a voltage source, the case of Fig.2 takes place. Such position of the center results from the known equivalent generator, when a voltage open circuit does not depend on R_2. If the two-pole shows the property of a current source, the center G is

posited in the second quadrant. Therefore, it is possible to accept that coordinates of the

point G define a generalized equivalent generator in Fig.3.

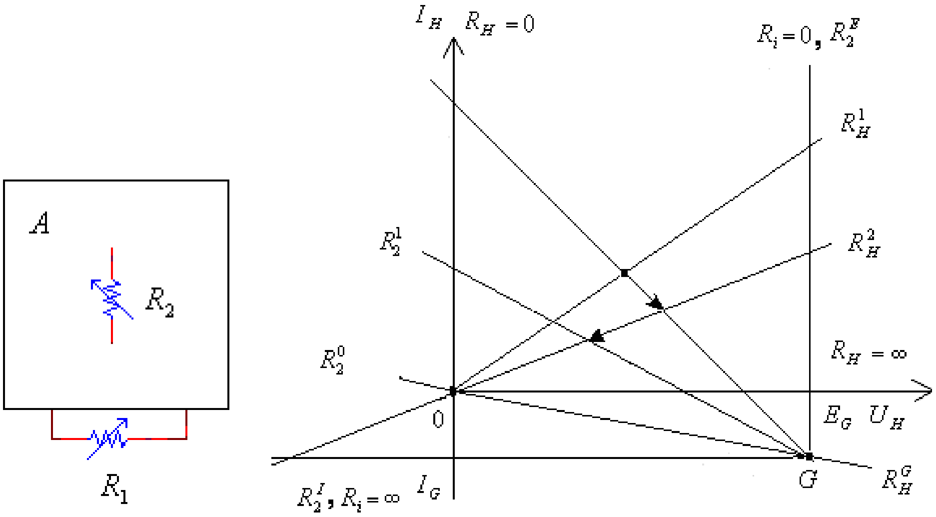
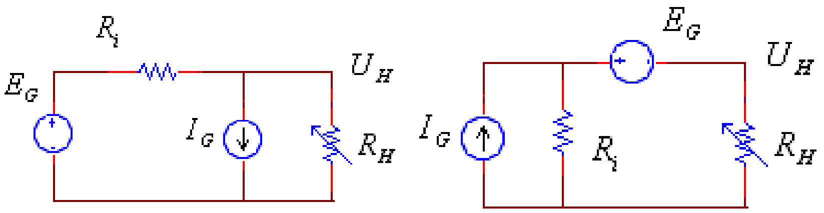


Fig.2. Active two-pole A and its characteristic family with changeable resistance R_2 and load R_1.

Besides the basic energy source of one kind (a voltage source E_G), there is an additional energy source of another kind (a current source I_G), that presents the corresponding theorem. The internal resistance R_i is determined by known rules.



(a) (b)

Fig.3. Generalized equivalent generator: a)- corresponds the voltage source; b)- corresponds the current source.

Physically, the bunch center G corresponds to such voltage E_G and load current I_G when the current through the element R_2 is equal to zero. The equation of the generalized equivalent or the load straight line is

$$I_H + I_G = -\frac{1}{R_i}(U_H - E_G).$$

Thus, a usual regime of an open circuit is not characteristic. The load R_H^G is the characteristic value in this case. In its turn, the resistance R_2^0 is the third characteristic value of R_2. Thus, these three values form the basic values R_2^E, R_2^I and the scale value R_2^0. Therefore, it is possible to express a running value and its change in a relative view through the corresponding cross ratio [3].

III. Generalized equivalent generator of an active two-port network

Let us consider an active two-port network with changeable conductivities of loads Y_{H1}, Y_{H2}

in Fig.4,a.

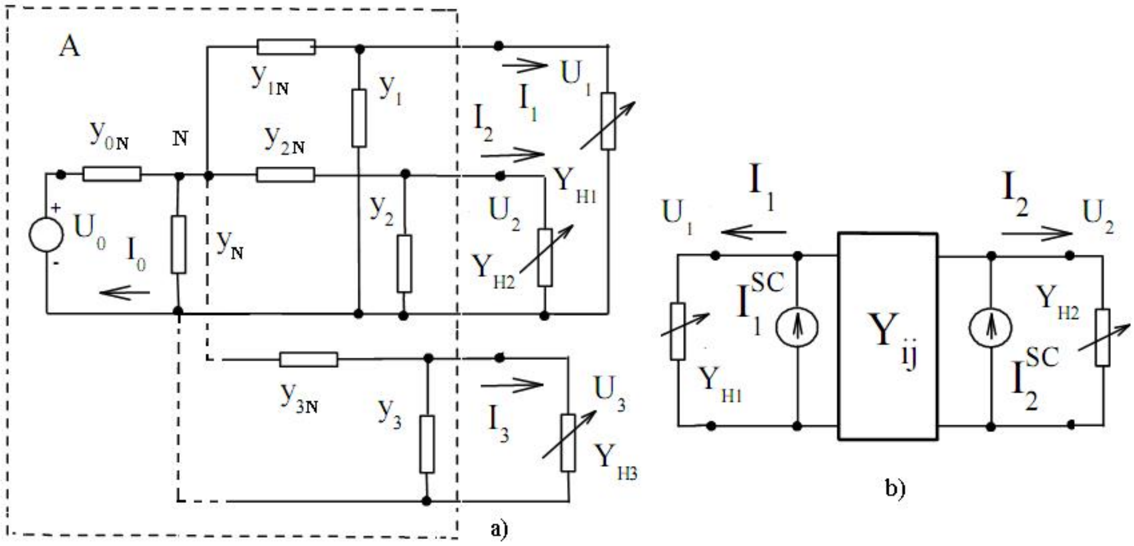


Fig.4. Active two-port (and multi-port) –a); traditional equivalent generator -b)

Taking into account the specified directions of currents, a network is described by the following system of the Y - parameters equations

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -Y_{11} & Y_{12} \\ Y_{12} & -Y_{22} \end{pmatrix} \cdot \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} + \begin{pmatrix} I_1^{SC} \\ I_2^{SC} \end{pmatrix}$$

where I_1^{SC}, I_2^{SC} are the short circuit SC currents $I_1^{SC} = Y_{10}U_0$, $I_2^{SC} = Y_{20}U_0$.

The expression shows that the active two-port network represents a passive part which is set by parameters of conductivity Y_{ij}, and two current generators I_1^{SC}, I_2^{SC}, Fig.4,b. We notice that the currents I_1^{SC}, I_2^{SC} are set by the parameters Y_{10}, Y_{20} which depend practically on all the elements of the two-port network, except y_1, y_2 .

Therefore, at possible changes, for example, of the conductivity y_N or y_{1N}, it is necessary to make the recalculation of values of these current generators that is inconvenient. The conductivity y_N can be a part of a possible third load. In this sense, the parameters of generalized equivalent generator, offered for an active two-pole, do not depend on such element of a circuit. Let us introduce, similarly, the generalized equivalent generator for the active two-port network on the basis of coordinates of the centers of bunches of straight lines [4].

Taking into account the voltages $U_1 = I_1 / Y_{H1}$, $U_2 = I_2 / Y_{H2}$, two bunches of load straight lines with parameters Y_{H1}, Y_{H2} are obtained in Fig.5,a.

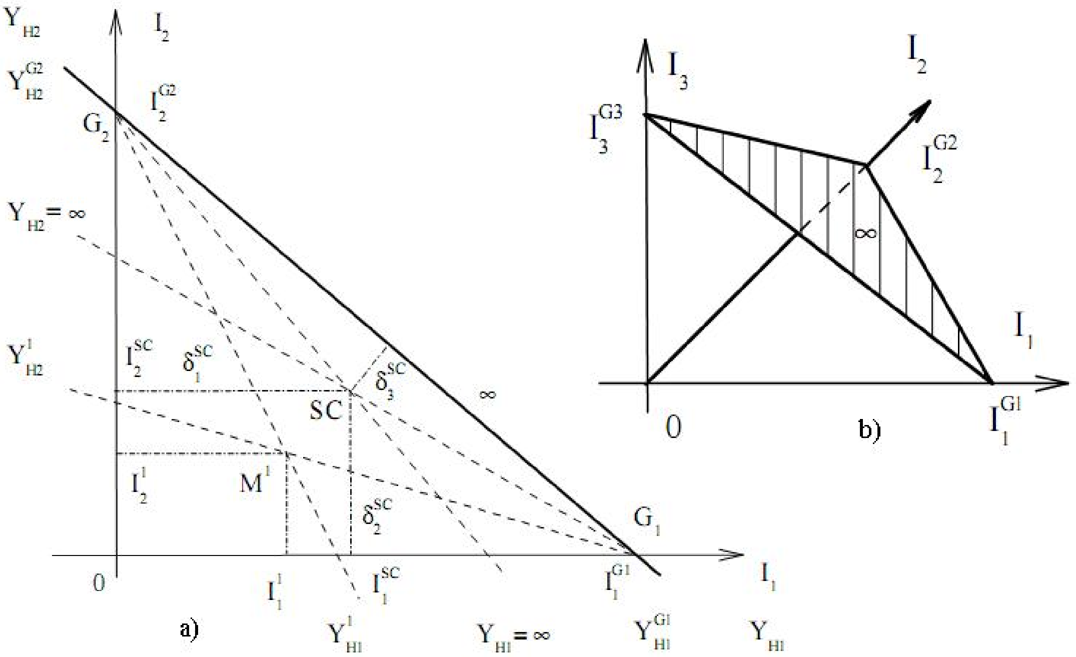


Fig.5. Two bunches of load straight lines with parameters Y_{H1}, Y_{H2} and position of the values I_1^{G1}, I_2^{G2} on the axes –a); position of the values $I_1^{G1}, I_2^{G2}, I_3^{G3}$ on the current axes -b)

The bunch center, the point G_2, corresponds to the bunch with the parameter Y_{H1} . The bunch center corresponds to such regime of the load Y_{H1} which does not depend on its value. It is carried out for the $I_1 = 0, U_1 = 0$ at the expense of a choice of regime parameters of the second load Y_{H2} . The parameters of the center G_1 of the bunch Y_{H2} are expressed similarly. Therefore, it is possible to accept that the coordinates of the points G_1, G_2 define the generalized equivalent generator in Fig.6.

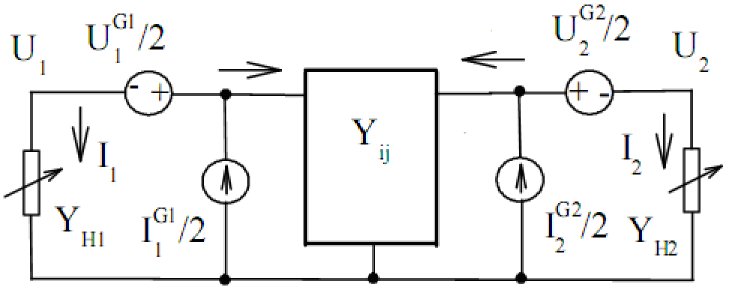


Fig.6. Generalized equivalent generator of an active two-port network

The parameters of voltage and current sources of the generalized equivalent generator of the active two-port network in Fig.6 do not depend on the conductivity y_N . It represents a practical interest. Therefore, only Y - parameters of the passive two-port network are recalculated. Further, the centre G_1 is defined by elements y_1, y_{1N} and does not depend on y_2, y_{2N} .Therefore, such a property of elements to simplify the circuit analysis.

IV. Generalized equivalent generator of an active multi-port network

Let us consider an active multi-port network with changeable conductivities of loads Y_{H1}, Y_{H2}, Y_{H3} in Fig.4,a. Then, three bunches of planes are obtained. The crossing of planes of one bunch among themselves defines a bunch axis. In turn, the points of intersection the bunch axes with current axes define the characteristic currents $I_1^{G1}, I_2^{G2}, I_3^{G3}$ in Fig.5,b. Therefore, it is possible to accept that the coordinates of the points $I_1^{G1}, I_2^{G2}, I_3^{G3}$ define the generalized equivalent generator in Fig.7 [5]. We notice, also, that the point I_1^{G1} is defined by the elements y_1, y_{1N} and does not depend on elements y_2, y_{2N}, y_3, y_{3N}.

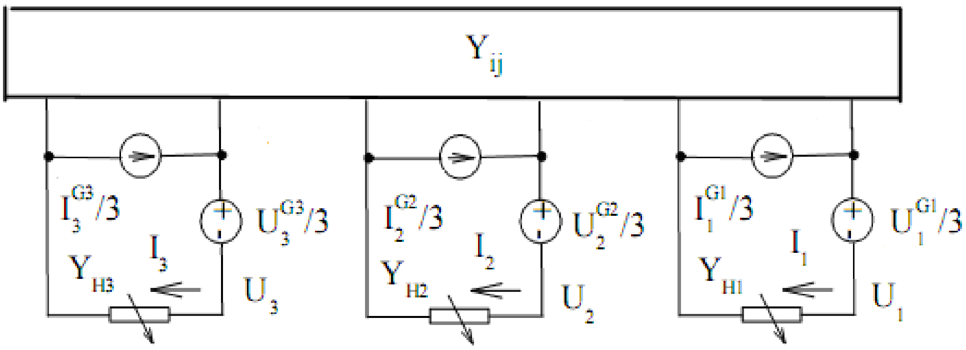


Fig.7. Generalized equivalent generator of an active multi-port network

V. Conclusions

The generalized equivalent generator of the active multi-port network in a form of the passive network and a set of the sources of current and voltage is proposed. The parameters of these sources do not depend on certain conductivities of the passive part. Such a property of the generator to simplify the circuit analysis.

VI. Reference

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