THE EXPERIMENTAL DETERMINATION OF THE MOMENT OF INERTIA

Radu-Vasile ARAMĂ, Andreea BÎRSAN, Andrian GÎRLEA*, Igor OVADENCO

Technical University of Moldova, Faculty of Computers, Informatics and Microelectronics, Department of Software Engineering and Automation, Group FAF-193, Chisinau, Republic of Moldova

*Autor correspondent: girlea.adrian@isa.utm.md

Abstract: *Accurate calculation of the moment of inertia of an irregular body is made difficult by the large number of quantities which must be measured. Trifilar suspensions are routinely used to measure the period of oscillation of the body in the horizontal plane. For accurate measurement the center of mass of the suspended object should be on the symmetry axis of the suspension. In this paper, we discuss the experimental determination of the moment of inertia with the help of the oscillation method. The procedure is described, the necessary calculations are derived and the relative importance of accuracy in different measurements is assessed. In addition to this we explore the error analysis in trifilar inertia measurements and the application of the moment of inertia method.*

Key words: *inertia, trifilar system, error, application, measurement.*

Introduction

Moment of inertia (or rotational inertia) is a tensor that describes the property of a rigid body to conserve the quality of rotational motion. In fact, it is the rotational analog of mass for linear motion. The moment of inertia must be specified with respect to a chosen axis of rotation. For a point mass, the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis. That point mass relationship becomes the basis for all other moments of inertia since any object can be built up from a collection of point masses.

Since the moment of inertia of a homogenous object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a point mass is defined by Eq. (1)

$$I = mr^2 \tag{1}$$

then the moment of inertia contribution by an infinitesimal mass element dm has the same form. This kind of mass element is called a differential element of mass and its moment of inertia is given by Eq. (2)

$$dI = r^2 dm \tag{2}$$

The moment of inertia of any other rigid body having a shape that can be described by a mathematical formula is commonly calculated by the integral calculus. For a body with a mathematically indescribable shape, the moment of inertia can be obtained by experiment. One of the experimental procedures employs the relation between the period (time) of oscillation of a torsion pendulum and the moment of inertia of the suspended mass. The time for one complete oscillation would depend on the stiffness of the wire and the moment of inertia of the disk; the larger the inertia, the longer the time.

The Trifilar Suspension System

Trifilar suspensions are used to measure the moments of inertia or gyration of solid bodies such as UAV (unmanned aerial vehicle), aircraft, boats, engines. For such measurements to be accurate the center of mass of the suspended body must be on the symmetry axis of the suspension. For irregularly shaped objects the precise location of the center of mass may be unknown and it becomes part of the measurements to determine its location. For large objects it may be impractical to precisely locate the center of mass on the symmetry axis in which case its displacement from the symmetry axis needs to be determined in order to correct the results (1).

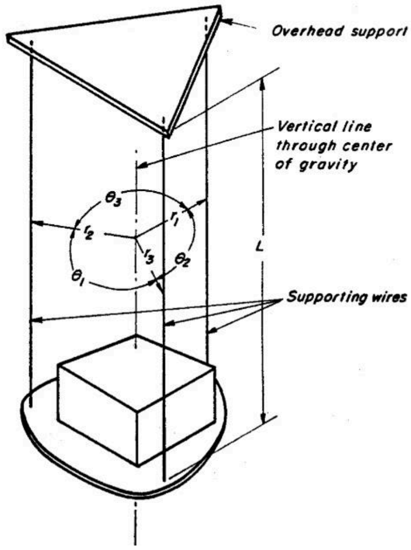

Figure 1. Experimental setup [1]

This experiment involves building and testing a trifilar pendulum. At its simplest, a trifilar pendulum is a platform rotating about a point via three evenly-spaced vertical strings attached to the outsides of the platform. When the disk is rotated, the strings are pulled to an angle and the disk is lifted very slightly (2). Then when it is released, the angled strings put a torque on the platform that turns it in the other direction, twisting it up a small distance on the other side of its equilibrium. Then it oscillates for a period of time as determined by damping. The period of this oscillation is proportional to the moment of inertia, such that the moment of inertia can be determined through measuring the period of oscillation.

These measurement systems are commonly used for determining moments of inertia in many fields such as the automotive or aerospace industries. In these situations, an object is put into a special frame such that it can be suspended about many different axes and then entire moment of inertia tensor can be measured. From the equations of motion, a relationship between the period of oscillation, mass of the system, and moment of inertia about the central axis can be made, Eq(3). This relationship can be used to measure the moment of inertia about the center of mass of any item placed on the disk.

$$I_{object} = \frac{R^2 g \tau^2}{4\pi^2 L}\left(M_{platform} + M_{object}\right) - I_{platform} \tag{3}$$

Where:

I refers to moment of inertia through center of mass about the z-axis of the system
R is the distance from the center of the disk to each string
g is acceleration due to gravity (approximated to by 9.8 $\mathrm{m/s^2}$)
τ is the period of oscillation
L is the length of the strings

Error Analysis in Trifilar Inertia Measurements

The trifilar system measures moments of inertia with consistently low errors in certain situations (2). When the ratio of mass to moment of inertia is low, the system is far more accurate. In addition, on very small masses, friction increases the error drastically. However, this error is reduced through increasing the amount of mass used.

Application of the moment of inertia method

Nowadays UAVs are used widely in civil and military applications. Missions like firefighting or detecting radiation levels around nuclear power sites are dangerous and hazardous to be performed by humans, but are obvious applications for UAVs. Increasingly other applications are being considered for economic reasons (3). As a result of this growth, accurate flight simulation is necessary to accomplish these sophisticated missions. In investigating the UAV dynamics and for developing a computational model to simulate the UAV's motion, it is necessary to determine moments of inertia accurately. This accuracy is crucial in a study of rotational flight, spinning for instance. Moments of inertia (and products of inertia) of a body indicate the resistance of the body to rotation, depending on the distribution of the mass and bodies with larger moments of inertia, resist more against rotation. There are two approaches to calculate these values; mathematical and experimental. In the theoretical-mathematical method the contribution made by all parts of the UAV has been considered. To calculate the moments of inertia, weight of all individual parts of the UAV and their distances to the UAV Centre of Gravity are required. Due to complexity of these calculations together with bearing in mind the fact that breaking down the already built UAV to its parts is not possible, make this approach impractical. On the other hand, the experimental approach proved itself as a suitable procedure specifically for small UAVs.

Conclusion

Moments of inertia play a vital role in the motion of rigid bodies. Accurate information for moments of inertia is, however, not easily obtained for common bodies such as vehicles, aero planes, trains and humans. Presented in this text is a procedure, requiring minimal equipment expenditure, for the determination of the moments of inertia. Trifilar suspensions are routinely used to measure the period of oscillation of the body in the horizontal plane.

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