

# EXCITABILITY AND COHERENCE RESONANCE OF DFB LASER WITH PASSIVE DISPERSIVE REFLECTOR

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## Abstract

We investigate the phenomena of excitability and coherence resonance of single mode DFB laser with passive dispersive reflector. We show that under an appropriate level of noise the excitable element can display the coherence resonance. Finally, we discuss the importance of excitability and coherence resonance for applications in communication networks.

## 1. Introduction

Excitability is a well established concept in biology [1, 3] and chemistry [4]. It can be illustrated in the biological context by the all-or-nothing behaviour of neurons; a sub-threshold stimulus implies only a local (i.e., non-propagated) response, while a stimulus above the threshold leads to a pulse propagating along the axon [5, 6]. However, excitability can also happen in optical systems and has been predicted to occur under certain conditions in a nonlinear ring cavity [7, 8], a laser with a saturable absorber [9], and a semiconductor laser subjected to delayed optical feedback [10]. Some general concepts of excitability in optics, the associated theoretical models, and the experimental evidence of excitability have been discussed in Ref. [11]. Convincing experimental evidence of excitability in a laser with a short external cavity is reported in [12]. In addition, an experimental investigation of the excitable properties of a solid-state laser with an intra-cavity saturable absorber (SA) is reported by Larotonda et al. [13]. Excitability in lasers attracts substantial interest because it offers prospects for practical applications in optoelectronic devices, primarily for optical switching, clock recovery, pulse reshaping, tunable pulses, and for generating a coherent resonance output pulse in communication networks.

Excitability originating from two main mechanisms can be found in literature [6]: so called class 1 excitability, when a saddle is close to equilibrium in the phase space, and class 2 excitability, when the stability of a rest state is lost through a Hopf bifurcation. In the saddle case, the excitability threshold corresponds to the stable manifold of the saddle. A sub-threshold-perturbation leads only to small local changes. A super threshold perturbation leads to a large excursion in phase space and all-or-none behaviour occurs. It has been demonstrated in Ref. [9] that a laser with a saturable absorber displays excitability due to an attractor (stable node) close to a saddle point. The same mechanism has been shown to pertain for excitability in a nonlinear ring cavity [8]. When the rest state is near the Hopf bifurcation

(class 2 excitability) the threshold is not so well defined and we speak instead of a threshold set, when the system can produce a spike having arbitrary intermediate amplitude [14]. The Hodgkin-Huxley model is one example of biological system, which does not have an all-or-none response. FitzHugh [1] referred to this as a quasi-threshold phenomenon.

The influence of noise is of particular interest for excitable systems. Noise can play an important role in the dynamics of semiconductor lasers. Many effects can be induced by noise but here we concentrate on coherence resonance. It is shown in Ref. [15] that additive noise can have quite different effects when acting on oscillatory, excitable, or bistable systems. The influence of noise in excitable or bistable systems is more dramatic than in an oscillatory one. For an excitable system under the influence of a small level of noise, the trajectory becomes almost periodic. This phenomenon is called coherence resonance. The first theoretical evidence of coherence resonance in an excitable FitzHugh-Nagumo system was given in Ref. [16], where it was shown that the coherence of noise induced oscillations is greatest for certain noise amplitude. This property of coherence resonance is explained by different noise-dependences of the activation and excursion times (the activation time is the time needed to excite the system from the stable point; while the excursion time is the time required to return from the excited state to the stable point). Ref. [9] presents theoretical evidence that coherence resonance can occur in a laser with a saturable absorber. The Lang-Kobayashi model of coherence resonance in a semiconductor laser with optical feedback is studied in Ref. [17]. More recently experimental evidence of coherence resonance in an optical system has been reported in Ref. [18], where the presence of the effect in the intensity of a laser diode with optical feedback is shown, and it is also demonstrated that both phase and amplitude fluctuations of the pulses play a significant role in the dynamics of the system.

In this paper we report studies of excitability and coherence resonance in a laser with a passive dispersive reflector (PDR). The paper is organized as follows. In Section 2, we present the equations of the device model. In Section 3, we demonstrate that excitability and coherence resonance occur in the proposed model. Conclusions are given in Section 4.

## 2. Laser setup and model

Figure 1 depicts the structure of the laser, which consists of one active DFB section, where a pump current is applied, and a completely passive dispersive reflector. The feedback comes from a Bragg grating that is integrated together with the laser within a compound and compact. The main differences to typical external mirrors are the extreme short separation between laser and reflector and the dispersion of the Bragg reflector. Similar devices are used for generating high-frequency single mode self-pulsations and serve very successful as optical clock in all-optical data regeneration [19]. The basic dispersive self-Q switching mechanism of these SP is well understood [20–23]. A single mode approximation of such devices was

proposed in Ref. [24] and used to discuss the self-pulsating behaviour. Subsequently, it was shown in Ref. [14] how the laser could be expected to exhibit excitability. Within this setup, we carry out a detailed analysis of the excitability and show that coherence resonance is also possible.

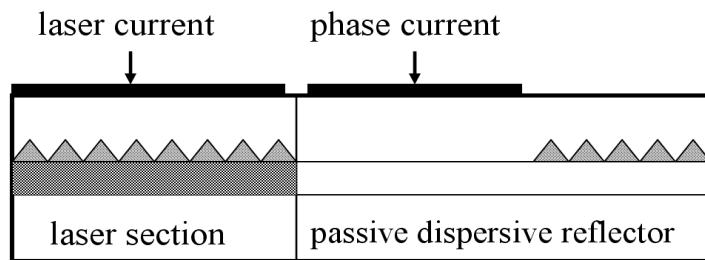


Figure 1. DFB laser with an integrated passive dispersive reflector.

We start from the rate equation used in [24]

$$\frac{dn}{dt} = J - n - (1+n)K(n)p, \quad (1)$$

$$\frac{dp}{dt} = TG(n)p, \quad (2)$$

where  $\tau$  is the dimensionless time, and  $n$  and  $p$  are the dimensionless carrier and photon numbers, respectively. The parameter  $J$  denotes the relative excess injection rate typically in the range  $1 < J < 10$  and  $T$  is the ratio between the carrier and photon lifetimes. The two functions  $K(n)$  and  $G(n)$  in equations (1)-(2) describe the influence of reflector on the laser dynamics and can be approximated by [24]

$$K(n) = K_0 + \frac{AW^2}{4(n-n_0)^2 + W^2}, \quad (3)$$

$$G(n) = n + \alpha\Delta n \tanh\left(\frac{n}{\Delta n}\right), \quad (4)$$

where  $A$ ,  $W$ ,  $K_0$ ,  $n_0$  are constants. The detuning  $n_0$  between the resonance peak of  $K(n)$  and the threshold density  $n = 0$  is considered as a branch parameter which can be varied. This variation can be achieved in the real devices by tuning the phase current (see Fig. 1).

### 3. Excitability and coherence resonance. Discussions

We begin by considering the stationary states of system (1)-(4). The system has two stationary solutions corresponding to laser “off” and “on”. The “on” state is  $n = 0$  and  $p = J/K(0)$ . When varying the detuning  $n_0$  and keeping all other parameters fixed, the resonance structure of  $K$  causes a dip of the stationary photon number  $p$  as shown in Fig. 2. Within a certain range on the left hand side of this dip, the stationary state becomes unstable and self-sustained pulsations emerge. A detailed stability and bifurcation analysis of this phenomenon has been presented elsewhere [14, 24]. On the right hand side of the power depletion, we found excitability for detuning  $n_0 = 0.005$  (point E).

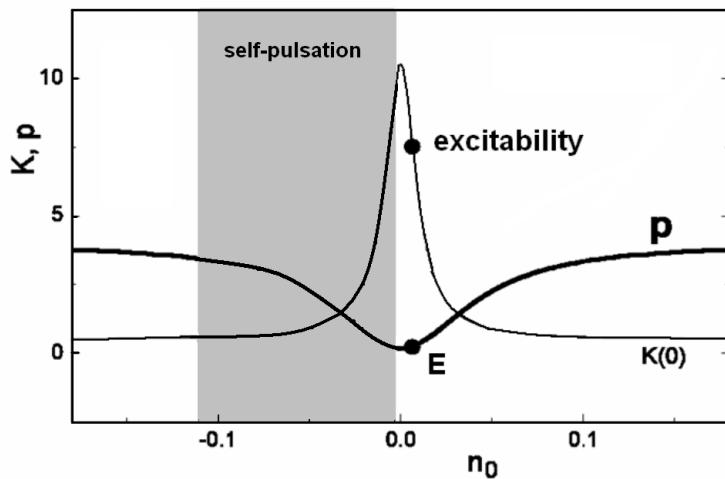


Figure 2. Dependence of the threshold value  $K(0)$  (thin solid) and of the stationary reduced photon number  $p = J = K(0)$  (thick solid) on the detuning parameter  $n_0$ . The dark region indicates the range of the detuning with self-pulsations. The full dots at  $n_0 = 0.005$  represent the detuning where excitability is demonstrated.

One motivation for research into excitability is its possible application in all-optical signal processing. We show some examples of the possible applications of excitability. As an optical limiter, the excitable element could serve to limit or suppress the noise level and any pulses below threshold. The excitable element can be also used as a logic element. For a sub-threshold input we have “0” output, but for an input above threshold the output is signal “1”. Consider sequences of pulses with different amplitudes and equal delay time between pulses, such as the case shown in Fig. 4. When the perturbation amplitudes are smaller than a threshold the response is negligible (see Fig. 4a). We observe a big response only to input pulses which exceed the threshold (see Fig. 4b). Other pulses cause negligible response and should be recognized as “0”-signals. Figure 4c shows the response of laser to a sequence of pulses with amplitudes above threshold. There is a substantial response to all input pulses. The selection criterion (position of threshold) can be varied by changing the parameters of the system. We mention that the numerical calculations show the existence of refractory period; i.e., when the two pulses are separated by a sufficiently short time interval the later pulse has insignificant effect on the system. Thus, the delay time between pulses has been selected to exceed the refractory period.

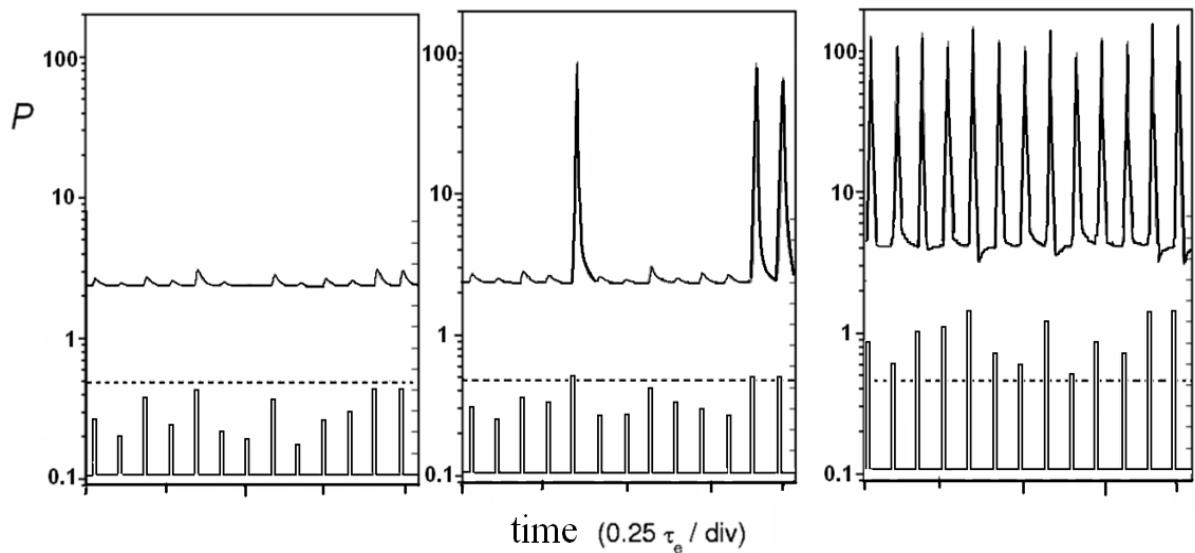
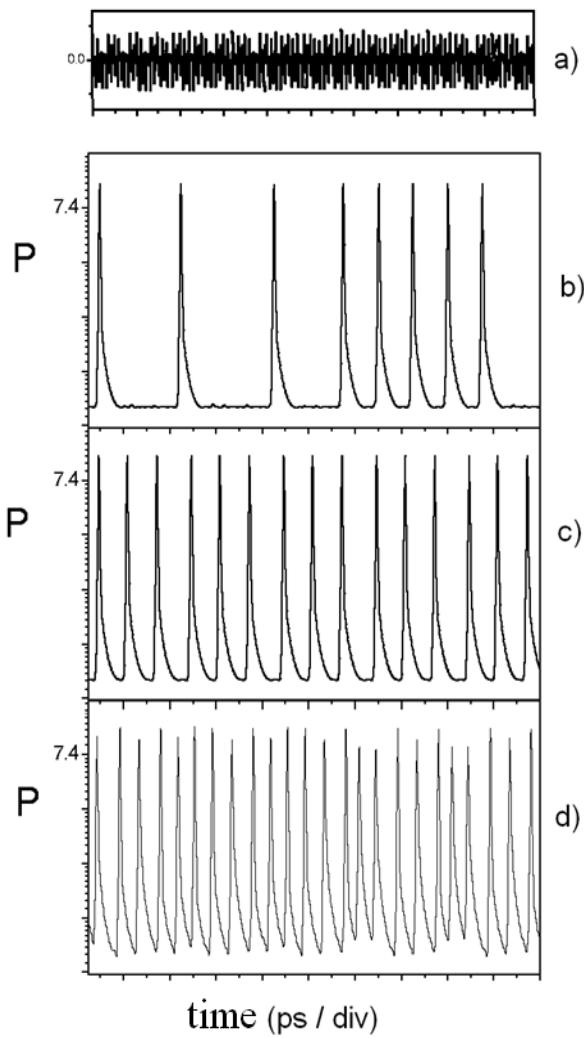


Figure 4. Response of the system to a train of perturbing pulses with different amplitudes. The dashed line indicates the threshold of the pulse amplitude.

Next we study the influence of noise on an excitable PDR laser. We consider the current fluctuations as a noise source (see top of Fig. 5). For this purpose, we added the term  $\xi(t)$  to the right hand side of equation (1), where  $\xi$  represents the external noise which is taken to be colored noise of the Ornstein-Uhlenbeck type Gaussianly distributed with zero mean and autocorrelation function

$$\langle \xi(t)\xi(t') \rangle = \frac{D}{\tau_c} \exp(-(t-t')/\tau_c).$$

We choose the parameter values  $A = 10$ ,  $W = 0.02$ ,  $\alpha = 0.05$ ,  $\Delta n = 0.05$ ,  $n_0 = 0.00025$ , and  $T = 250$  so that the laser operates in the excitable regime. When noise is absent the output laser photon number is constant. When noise is introduced into the system ( $D=0.05$ ) random output pulses are observable (see Fig. 5b) for the noise correlation time  $\tau_c = 0.3$  and the power spectrum has broad features.



When the parameter  $D$  is increased the spikes become more frequent and for  $D = 0.075$  the behavior becomes almost periodic as suggested by Fig. 5c. Further increase of noise level leads to irregular output signal (Fig. 5d). From these results, we conclude that there exists optimal amplitude of external noise for which the output photon number is almost periodic. It is apparent from Figure 5 that increasing of the noise level affects not only the duration of the pulses but also their amplitudes. This result confirms the idea that the irregularity of the pulse amplitude increases with noise. Thus, we predict that a laser with an integrated dispersive reflector can display coherence resonance. If such an effect could be produced experimentally it may be applied in network communications where the enhancement of regular pulses is required. The pulses could be obtained as a result of the interaction of the PDR laser with the noise source.

Figure 5. Coherence resonance induced by an appropriate noise level: (a) Noise shape; (b) Laser response for  $D=0.05$ ; (c) Periodic response (coherence resonance) for  $D=0.075$ ; (d) Irregular output resulting from a high noise level  $D=0.15$ .

#### 4. Conclusions

We have carried out an investigation of the dynamics of single mode semiconductor laser with passive dispersive reflector. It has been shown that a laser with passive dispersive reflector can display the phenomenon of excitability. We have discussed the possible application of an excitable element in all-optical signal processing as an optical limiter to limit or suppress the noise level and any pulses below threshold. The excitable element can be also used as a logic element. For a sub-threshold input we have “0” output, but for an input above threshold the output is signal “1”. We have also demonstrated that an excitable laser can show the phenomenon of coherence resonance. Low level noise applied to the excitable system results in a sequence of random output pulses. However, when the noise level is increased to an appropriate value, the response becomes almost periodic in a manner characteristic of coherence resonance. Further increase of the noise level leads to irregular output signal. It may be concluded that the considered type of excitable system could be potentially applied as a functional optoelectronic element.

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